Extending the Ideas

A function that is bounded above has an infinite number of upper bounds, but there is always a *least upper bound*, i.e., an upper bound that is less than all the others. This least upper bound may or may not be in the range of f. For each of the following functions, find the least upper bound and tell whether or not it is in the range of the function.

(a) $f(x) = 2 - 0.8x^2$

(b) $g(x) = \dfrac{3x^2}{3 + x^2}$

(c) $h(x) = \dfrac{1 - x}{x^2}$

(d) $p(x) = 2 \sin(x)$

(e) $q(x) = \dfrac{4x}{x^2 + 2x + 1}$

84. Writing to Learn A continuous function f has domain all real numbers. If $f(-1) = 5$ and $f(1) = -5$, explain why f must have at least one zero in the interval $[-1, 1]$. (This generalizes to a property of continuous functions known as the Intermediate Value Theorem.)

85. Proving a Theorem Prove that the graph of every odd function with domain all real numbers must pass through the origin.

86. Finding the Range Graph the function $f(x) = \dfrac{3x^2 - 1}{2x^2 + 1}$ in the window $[-6, 6]$ by $[-2, 2]$.

(a) What is the apparent horizontal asymptote of the graph?

(b) Based on your graph, determine the apparent range of f.

(c) Show algebraically that $-1 \le \dfrac{3x^2 - 1}{2x^2 + 1} < 1.5$ for all x, thus confirming your conjecture in part (b).