Equivalent Representations for Intervals of Real Numbers

Open or Closed?

Interval Notation

$(-\infty, \infty)$

Do Now: Complete the other 3 representations

Number Line

-5 -4 -3 -2 -1 0 1 2 3 4 5

Inequality Notation

$x \in \mathbb{R}$

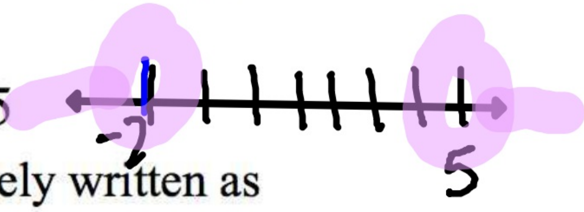
(is an element) of the real #s

Verbal Description

x is any real number between $-\infty$ and ∞

How do I write a separated inequality in interval notation?

- When combining intervals, we use the symbol U to indicate that we are including both intervals.
- Example: x < -2 or x > 5



  - The intervals are separately written as
    - $(-\infty, -2)$ and $(5, \infty)$
  - We write this combine as
    - $(-\infty, -2)\cup(5, \infty)$

union "and"

Objectives

- **Solve absolute value inequalities** and categorize solutions as ‘and’ or ‘or’ inequalities using a graphic organizer.
- Success Criteria
  - Understand why there is a case 1 and case 2 for absolute value inequalities
  - Graph solutions on a number line using test points
  - Rewrite in inequality and interval notation
- Vocabulary: absolute value, test points

Absolute Value of a Real Number

The absolute value of a real number a

$$|a| = \begin{cases} a & \text{if } a \ge 0 \\ -a & \text{if } a < 0 \end{cases}$$

example

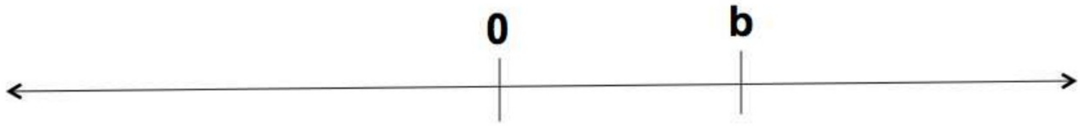
$|23| = 23$ $|0| = 0$

$|-304| = -(-304) = 304$

Remember: The purpose of the absolute value function is to make a number positive or keep a number positive or zero.

Case 1

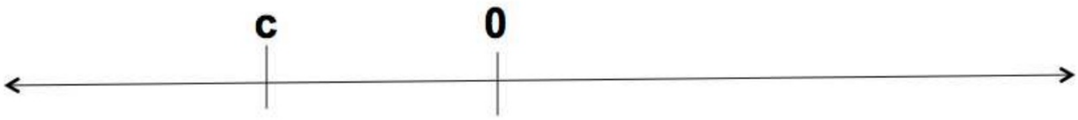
- Consider the following number, b



- b $>$ 0
- $|b| = b$

Case 2

- Consider the following number, c



- c $<$ 0
- $|c| = -c$

Science Connection

- When we take the absolute value of a number, we are finding the magnitude of the number. When there are only positive values in the scientific quantity, we call it a scalar. When there are negative values, we call it a vector.
- For example, the velocity of an object can be both positive and negative (vector), but the speed (scalar) is the absolute value of the velocity, since you can't have a negative speed.

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Why should I care about the absolute value of a real number anyway?

Applied Mathematicians spend a lot of their time trying to come up with ways to describe the real world with mathematical models. A precise common notation makes modeling, solving and communicating problems easier.

Scenario A: You complain to your classmates about how long the bus trip to school takes every morning. After listening to your complaints a student asks, "How far do you live from school?" You reply, "15 miles."

Scenario B: You complain to your classmates about how long the bus trip to school takes every morning. After listening to your complaints, a student asks, "How far do you live from school?" You reply, "15 miles to the south."

THINK-PAIR-SHARE: Use what you know about number lines, and the absolute value function to model Scenario A and B. Discuss your ideas with your group. Come to a consensus.

Scenerio A:

Scenerio B:

A)

B)

Why should I care about the absolute value of a real number anyway?

Scenario A: You complain to your classmates about how long the bus trip to school takes every morning. After listening to your complaints a student asks, "How far do you live from school?" You reply, "15 miles."

When you respond fifteen miles from school, you are giving the absolute value or magnitude of your trip distance $|d|=15$, because we don't have any direction.

Scenario B: You complain to your classmates about how long the bus trip to school takes every morning. After listening to your complaints, a student asks, "How far do you live from school?" You reply, "15 miles to the south."

When you respond fifteen miles to the south, you are giving a directed distance that can be modeled with a signed number for example the standard solution would be to use -15 for distances to the south and +15 for distances to the north.

Properties of Absolute Value

Let a and b be real numbers.	**Example**
1. $\lvert a \rvert \geq 0$	$\lvert 23 \rvert = 23 \geq 0$
2. $\lvert -a \rvert = \lvert a \rvert$	$\lvert -\sqrt{5} \rvert = \lvert \sqrt{5} \rvert = \sqrt{5}$
3. $\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$	$\left\lvert -12\left(\frac{3}{5}\right) \right\rvert = \lvert -12 \rvert \left\lvert \frac{3}{5} \right\rvert$
4. $\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a \rvert}{\lvert b \rvert},\ b \neq 0$	$\left\lvert \frac{-1.5}{303} \right\rvert = \frac{\lvert -1.5 \rvert}{\lvert 303 \rvert}$

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AM: Solve absolute value eqns with 1-variable

$$\left|\frac{6}{7}x+8\right| = 9$$

1. Identify the solutions of the equation: $\left|\frac{6}{7}x+8\right|+3=12$

$-3 \quad -3$

[A] $\left\{\frac{7}{6}, -\frac{119}{6}\right\}$ [B] $\left\{\frac{161}{6}, \frac{119}{6}\right\}$ [C] $\left\{\frac{6}{7}, -\frac{161}{6}\right\}$ [D] $\left\{\frac{119}{6}, -\frac{102}{7}\right\}$

Case 1 (non-negative)

a = $\frac{6}{7}x+8$

0 a

a $\geq$ 0

|a| = a

$\frac{6}{7}x+8=9$

$-8 \quad -8$

$\frac{7}{6}\cdot\frac{6}{7}x = 1\cdot\frac{7}{6}$

$\boxed{x=\frac{7}{6}}$

Case 2 (negative)

a = $\frac{6}{7}x+8$

a 0

a < 0

|a| = $-a$

$-\left(\frac{6}{7}x+8\right)=9$

$-\frac{6}{7}x-8=9$

$+8 \quad +8$

$-\frac{7}{6}\cdot-\frac{6}{7}x=17\cdot-\frac{7}{6}$

$\boxed{x=\frac{-119}{6}}$