# Properties of Radicals

Let $u$ and $v$ be real numbers, variables, or algebraic expressions, and $m$ and $n$ be positive integers greater than 1. We assume that all of the roots are real numbers and all of the denominators are not zero.

| | Property | Example |
|---|---|---|
| 1. | $\sqrt[n]{uv} = \sqrt[n]{u} \cdot \sqrt[n]{v}$ | $\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$ |
| 2. | $\sqrt[n]{\dfrac{u}{v}} = \dfrac{\sqrt[n]{u}}{\sqrt[n]{v}}$ | $\dfrac{\sqrt[4]{96}}{\sqrt[4]{6}} = \sqrt[4]{\dfrac{96}{6}} = \sqrt[4]{16} = 2$ |
| 3. | $\sqrt[m]{\sqrt[n]{u}} = \sqrt[m \cdot n]{u}$ | $\sqrt[2]{\sqrt[3]{7}} = \sqrt[6]{7}$ |
| 4. | $\left(\sqrt[n]{u}\right)^n = u$ | $\left(\sqrt[4]{5}\right)^4 = 5$ |
| 5. | $\sqrt[n]{u^m} = \left(\sqrt[n]{u}\right)^m$ | $\sqrt[3]{27^2} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$ |
| 6. | $\sqrt[n]{u^n} = \begin{cases} \lvert u \rvert & n \text{ is even} \\ u & n \text{ is odd} \end{cases}$ | $\sqrt{(-6)^2} = 6$ <br> $\sqrt[3]{(-6)^3} = -6$ |

# AM: Prime Factorization

**1.** Write 126 as a product of primes.

[A] $2^2 \cdot 3 \cdot 7$ [B] $2 \cdot 3^2 \cdot 5$ [C] $2^2 \cdot 3^2 \cdot 7$ [D] none of these

$$\begin{array}{r|l} 2 & 126 \\ 3 & 63 \\ 3 & 21 \\ & 7 \end{array}$$

$$2 \cdot 3^2 \cdot 7$$

**Note: Exponential Notation is a convenient and mathematically Sound system for expressing repeated multiplication.**

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# AM: Prime Factorization

**2.** Which shows 1800 as a product of primes?

[A] $4 \times 3^2 \times 5^2$ [B] $2 \times 3 \times 5$ [C] $2^3 \times 3^2 \times 5^2$ [D] $2 \times 3^2 \times 5^2$

$$
\begin{array}{r|l}
2 & 1800 \\ \hline
2 & 900 \\ \hline
2 & 450 \\ \hline
3 & 225 \\ \hline
3 & 75 \\ \hline
5 & 25 \\ \hline
 & 5
\end{array}
$$

$2^3 \cdot 3^2 \cdot 5^1$

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# AM: Cube and Fourth Roots

Simplify: $= \sqrt[3]{2^6} = \sqrt[3]{(2^2)^3} = 2^2 = 4$

**1.** $\sqrt[3]{64}$ [A] 4 [B] 16 [C] 5 [D] 21 R 1

$$\begin{array}{r|l} 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array} = 2^6$$

**2.** $\sqrt[4]{81}$

LO: The __________ root of __________ is __________ because __________— is equal to ________.

LO: The __________ root of __________ is __________ because __________— is equal to ________.

HINT: If you don't see the pattern use prime factorization!

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# AM: Cube and Fourth Roots

**3.** $\sqrt[3]{-8}$ [A] 2 [B] $-2$ [C] 4 [D] not a real number

**4.** $\sqrt[3]{-216}$

**What properties of radicals did you use?**

$$\begin{array}{r|l} -2 & 8 \\ \hline -2 & 4 \\ \hline & -2 \end{array}$$

$(-2)^3 = 8$

$\sqrt[3]{(-2)^3}$

$= -2$

LO: The __________ _______ of _________ is _________ because __________—— is equal to ________.

LO: The __________ _______ of _________ is _________ because __________—— is equal to ________.

HINT: If you don't see the pattern use prime factorization!

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## AM: Simplify nth Roots

Simplify:

**1.** $\sqrt[3]{48}$ [A] $2\sqrt[3]{6}$ [B] $6\sqrt[3]{2}$ [C] $4\sqrt[3]{12}$ [D] $2\sqrt[3]{12}$

$$\begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$2^4 \cdot 3$

$$\sqrt[3]{2^4 \cdot 3} = \sqrt[3]{2^3} \cdot 2 \cdot 3$$

$$= 2\sqrt[3]{6}$$

HINT: If you don't see the pattern use prime factorization!

# AM: Simplify nth Roots

**2.** $\left(\sqrt[3]{x^6}\right)^3$ [A] $x^6$ [B] $x^{3/2}$ [C] $x^{1/2}$ [D] $x^{18}$

Justify your answer with a Property of Radicals.
State your answer as a complete sentence.

$$\left(\sqrt[3]{x^6}\right)^3 = x^6$$

LO: **The quantity**, cube root of the **6th power of x**, cubed is equivalent to ________, by the _________ Property of Radicals.

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# AM: Simplify nth Roots

**3.** $\sqrt[4]{x^{15}y^{11}}$

[A] $x^3y^2\sqrt{x^3y^3}$ [B] $x^3y^3\sqrt[4]{x^3y^2}$ [C] $x^{11}y^7\sqrt{xy}$ [D] $x^3y^2\sqrt[4]{x^3y^3}$

Simplify. Show all work.
Justify your answer with the Properties of Radicals.
State your answer as a complete sentence.

$$\sqrt[4]{\underline{x^{12}}\cdot x^3\cdot \underline{y^8}\cdot y^3} = x^3y^2\sqrt[4]{x^3y^3}$$

LO: The **fourth root** of the **product of** x to the fifteenth and y to the eleventh is **equivalent to** ________, by the ____________ _________________Properties of Radicals.

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## AM: Rationalizing denominators

Rationalize the denominator:

**1.** $\dfrac{\sqrt{15}}{\sqrt{5q}}$ [A] $\dfrac{\sqrt{3q}}{q}$ [B] $\dfrac{\sqrt{3}}{q}$ [C] $\dfrac{\sqrt{15}}{5q}$ [D] $\sqrt{15}$

**There are multiple ways to begin this problem. The goal is to eliminate radicals from the denominator.**

$$\sqrt{\frac{15}{5q}} = \sqrt{\frac{3}{q}} = \frac{\sqrt{3}}{\sqrt{q}} \cdot \boxed{\frac{\sqrt{q}}{\sqrt{q}}} = \frac{\sqrt{3q}}{\sqrt{q^2}}$$

identity

# AM: Rationalizing denominators

**2.** $\dfrac{\sqrt{108x^7y}}{\sqrt{3x^5y^2}}$ [A] $\dfrac{\sqrt{36x^2}}{\sqrt{y}}$ [B] $\dfrac{\sqrt{324x^{12}y^3}}{\sqrt{3x^5y^2}}$ [C] $\dfrac{36x^4}{y}$ [D] $\dfrac{6x\sqrt{y}}{y}$

$$\sqrt{\frac{108x^7y}{3x^5y^2}} \Rightarrow \sqrt{36x^2y^{-1}} = \frac{\sqrt{36x^2}}{\sqrt{y}} = \frac{6x}{\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}}$$

$$\frac{6x\sqrt{y}}{y}$$

## Definition: Rational Exponents

Let $u$ be a real number, variable, or algebraic expression, and $n$ an integer greater than 1, then

$$u^{1/n} = \sqrt[n]{u}.$$

$\sqrt[3]{16} = 16^{1/3}$

If $m$ is a positive integer, $\dfrac{m}{n}$ is in reduced form, and all roots real numbers, then

$$u^{m/n} = \left(u^{1/n}\right)^m = \left(\sqrt[n]{u}\right)^m \text{ and } u^{m/n} = \left(u^m\right)^{1/n} = \sqrt[n]{u^m}$$

# AM: Write Roots as Exponential Expressions

**1.** Write $\sqrt{6y}$ in exponential form.

[A] $(6y)^{1/2}$  [B] $(6y)^{1/4}$  [C] $(6y)^{2}$  [D] $6y^{2}$

$(6y)^{1/2}$

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# AM: Write Roots as Exponential Expressions

**2.** Express using fractional exponents: $\sqrt[9]{c^5}$

[A] $5^{9/c}$ [B] $c^{9/5}$ [C] $c^{5/9}$ [D] $c^{1/45}$

$$\sqrt[9]{c^5} = c^{5/9}$$

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