P.2
Cartesian Coordinate System

What you'll learn about
- Cartesian Plane
- Absolute Value of a Real Number
- Distance Formulas
- Midpoint Formulas
- Equations of Circles
- Applications

. . . and why
These topics provide the foundation for the material that will be covered in this textbook.

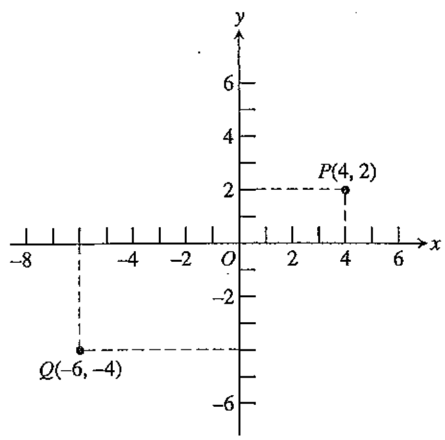


FIGURE P.6 The Cartesian coordinate plane.

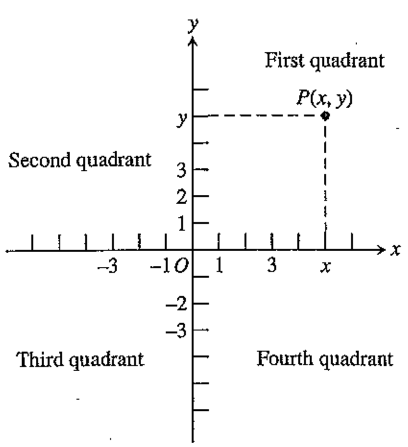


FIGURE P.7 The four quadrants. Points on the x- or y-axis are not in any quadrant.

Cartesian Plane

The points in a plane correspond to ordered pairs of real numbers, just as the points c a line can be associated with individual real numbers. This correspondence creates tł **Cartesian plane**, or the **rectangular coordinate system** in the plane.

To construct a rectangular coordinate system, or a Cartesian plane, draw a pair of pe pendicular real number lines, one horizontal and the other vertical, with the line intersecting at their respective 0-points (Figure P.6). The horizontal line is usually tł **x-axis** and the vertical line is usually the **y-axis**. The positive direction on the x-ax is to the right, and the positive direction on the y-axis is up. Their point of intersectio O, is the **origin of the Cartesian plane.**

Each point P of the plane is associated with an **ordered pair (x, y)** of real numbers, **(Cartesian) coordinates of the point**. The **x-coordinate** represents the intersectio the x-axis with the perpendicular from P, and the **y-coordinate** represents the intersec of the y-axis with the perpendicular from P. Figure P.6 shows the points P and Q v coordinates $(4, 2)$ and $(-6, -4)$, respectively. As with real numbers and a number line use the ordered pair (a, b) for both the name of the point and its coordinates.

The coordinate axes divide the Cartesian plane into four **quadrants**, as showı Figure P.7.

EXAMPLE 1 Plotting Data on U.S. Exports to Mexico

The value in billions of dollars of U.S. exports to Mexico from 1996 to 2003 is gi in Table P.2. Plot the (year, export value) ordered pairs in a rectangular coordiı system.

Table P.2 U.S. Exports to Mexico

Year	U.S. Exports (billions of dollars)
1996	56.8
1997	71.4
1998	78.8
1999	86.9
2000	111.3
2001	101.3
2002	97.5
2003	97.4

Source: U.S. Census Bureau, Statistical Abstract of the United States, 2001, 2004–2005.

SOLUTION

The points are plotted in Figure P.8 on page 15.

Now try Exercise 3

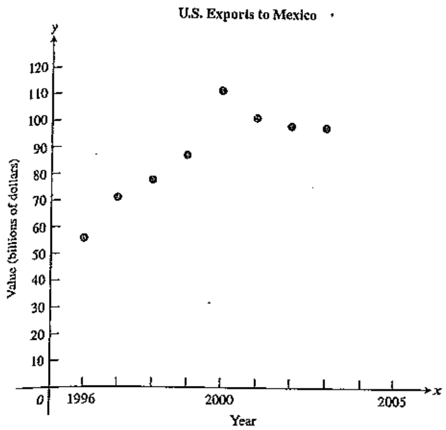


FIGURE P.8 The graph for Example 1.

A **scatter plot** is a plotting of the (x, y) data pairs on a Cartesian plane. Figure P.8 shows a scatter plot of the data from Table P.2.

Absolute Value of a Real Number

The *absolute value of a real number* suggests its **magnitude** (size). For example, the absolute value of 3 is 3 and the absolute value of -5 is 5.

DEFINITION Absolute Value of a Real Number

The **absolute value of a real number** a is

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0. \\ 0, & \text{if } a = 0 \end{cases}$$

EXAMPLE 2 Using the Definition of Absolute Value

Evaluate:

(a) $|-4|$ **(b)** $|\pi - 6|$

SOLUTION

(a) Because $-4 < 0$, $|-4| = -(-4) = 4$.

(b) Because $\pi \approx 3.14$, $\pi - 6$ is negative, so $\pi - 6 < 0$. Thus, $|\pi - 6| = -(\pi - 6) = 6 - \pi \approx 2.858$.

Now try Exercise 9.

Here is a summary of some important properties of absolute value.

Properties of Absolute Value

Let a and b be real numbers.

1. $|a| \geq 0$ **2.** $|-a| = |a|$

3. $|ab| = |a||b|$ **4.** $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, b \neq 0$

Distance Formulas

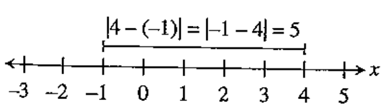


FIGURE P.9 Finding the distance between -1 and 4.

The *distance* between -1 and 4 on the number line is 5 (see Figure P.9). This distance may be found by subtracting the smaller number from the larger: $4 - (-1) = 5$. If we use absolute value, the order of subtraction does not matter: $|4 - (-1)| = |-1 - 4| = 5$.

Distance Formula (Number Line)

Let a and b be real numbers. The **distance between a and b** is

$$|a - b|.$$

Note that $|a - b| = |b - a|$.

ABSOLUTE VALUE AND DISTANCE

If we let $b = 0$ in the distance formula we see that the distance between a and 0 is $|a|$. Thus, the absolute value of a number is its distance from zero.

To find the *distance* between two points that lie on the same horizontal or vertical l in the Cartesian plane, we use the distance formula for points on a number line. example, the distance between points x_1 and x_2 on the x-axis is $|x_1 - x_2| = |x_2 -$ and the distance between the points y_1 and y_2 on the y-axis is $|y_1 - y_2| = |y_2 - y_1$

To find the distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ that do not lie on same horizontal or vertical line we form the right triangle determined by P, Q, a $R(x_2, y_1)$ (Figure P.10).

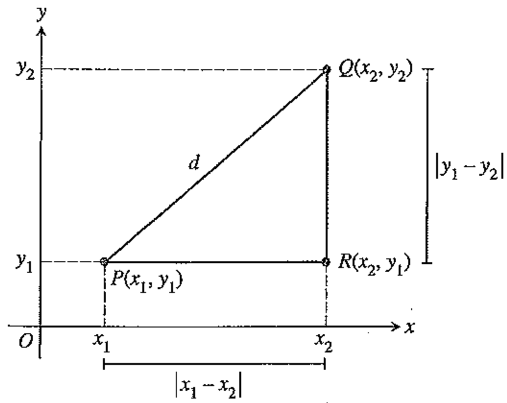


FIGURE P.10 Forming a right triangle with hypotenuse $\overline{PQ}$.

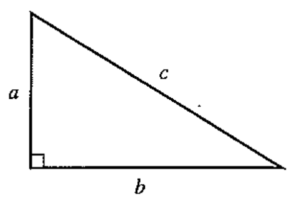


FIGURE P.11 The Pythagorean theorem: $c^2 = a^2 + b^2$.

The distance from P to R is $|x_1 - x_2|$, and the distance from R to Q is $|y_1 - y_2|$. By **Pythagorean Theorem** (see Figure P.11), the distance d between P and Q is

$$d = \sqrt{|x_1 - x_2|^2 + |y_1 - y_2|^2}.$$

Because $|x_1 - x_2|^2 = (x_1 - x_2)^2$ and $|y_1 - y_2|^2 = (y_1 - y_2)^2$, we obtain the follow formula.

Distance Formula (Coordinate Plane)

The **distance d between points $P(x_1, y_1)$ and $Q(x_2, y_2)$** in the coordinate plane is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

EXAMPLE 3 Finding the Distance Between Two Points

Find the distance d between the points (1, 5) and (6, 2).

SOLUTION

$$\begin{aligned} d &= \sqrt{(1 - 6)^2 + (5 - 2)^2} && \text{The distance formula} \\ &= \sqrt{(-5)^2 + 3^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \approx 5.83 && \text{Using a calculator} \end{aligned}$$

Now try Exercise 1

Midpoint Formulas

When the endpoints of a segment in a number line are known, we take the average of their coordinates to find the midpoint of the segment.

Midpoint Formula (Number Line)

The **midpoint of the line segment with endpoints *a* and *b*** is

$$\frac{a+b}{2}.$$

EXAMPLE 4 Finding the Midpoint of a Line Segment

The midpoint of the line segment with endpoints -9 and 3 on a number line is

$$\frac{(-9)+3}{2} = \frac{-6}{2} = -3.$$

See Figure P.12. *Now try Exercise 23.*

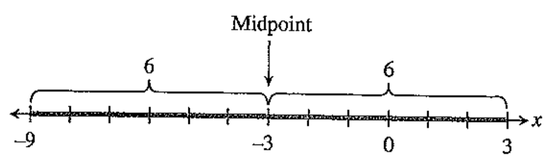


FIGURE P.12 Notice that the distance from the midpoint, -3, to 3 or to -9 is 6. (Example 4)

Just as with number lines, the midpoint of a line segment in the coordinate plane is determined by its endpoints. Each coordinate of the midpoint is the average of the corresponding coordinates of its endpoints.

Midpoint Formula (Coordinate Plane)

The **midpoint of the line segment with endpoints** (a, b) **and** (c, d) is

$$\left(\frac{a+c}{2}, \frac{b+d}{2}\right).$$

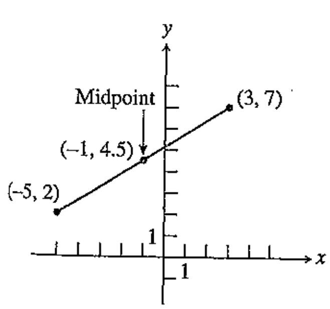


FIGURE P.13 (Example 5.)

EXAMPLE 5 Finding the Midpoint of a Line Segment

The midpoint of the line segment with endpoints $(-5, 2)$ and $(3, 7)$ is

$$(x, y) = \left(\frac{-5+3}{2}, \frac{2+7}{2}\right) = (-1, 4.5).$$

See Figure P.13. *Now try Exercise 25.*

QUICK REVIEW P.2

In Exercises 1 and 2, plot the two numbers on a number line. Then find the distance between them.

1. $\sqrt{7}, \sqrt{2}$ **2.** $-\frac{5}{3}, -\frac{9}{5}$

In Exercises 3 and 4, plot the real numbers on a number line.

3. $-3, 4, 2.5, 0, -1.5$ **4.** $-\frac{5}{2}, -\frac{1}{2}, \frac{2}{3}, 0, -1$

In Exercises 5 and 6, plot the points.

5. $A(3, 5), B(-2, 4), C(3, 0), D(0, -3)$

6. $A(-3, -5), B(2, -4), C(0, 5), D(-4, 0)$

In Exercises 7–10, use a calculator to evaluate the expression. Round your answer to two decimal places.

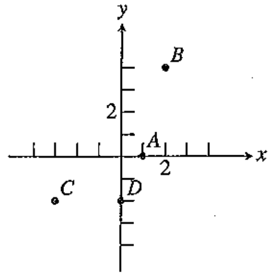
7. $\frac{-17 + 28}{2}$ **8.** $\sqrt{13^2 + 17^2}$

9. $\sqrt{6^2 + 8^2}$ **10.** $\sqrt{(17 - 3)^2 + (-4 - 8)^2}$

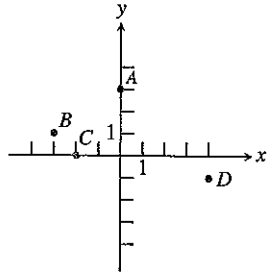
SECTION P.2 EXERCISES

In Exercises 1 and 2, estimate the coordinates of the points.

1.



2.



In Exercises 3 and 4, name the quadrants containing the points.

3. (a) $(2, 4)$ **(b)** $(0, 3)$ **(c)** $(-2, 3)$ **(d)** $(-1, -4)$

4. (a) $\left(\frac{1}{2}, \frac{3}{2}\right)$ **(b)** $(-2, 0)$ **(c)** $(-1, -2)$ **(d)** $\left(-\frac{3}{2}, -\frac{7}{3}\right)$

In Exercises 5–8, evaluate the expression.

5. $3 + |-3|$ **6.** $2 - |-2|$

7. $|(-2)3|$ **8.** $\frac{-2}{|-2|}$

In Exercises 9 and 10, rewrite the expression without using absolute value symbols.

9. $|\pi - 4|$ **10.** $|\sqrt{5} - 5/2|$

In Exercises 11–18, find the distance between the points.

11. $-9.3, 10.6$ **12.** $-5, -17$

13. $(-3, -1), (5, -1)$ **14.** $(-4, -3), (1, 1)$

15. $(0, 0), (3, 4)$ **16.** $(-1, 2), (2, -3)$

17. $(-2, 0), (5, 0)$ **18.** $(0, -8), (0, -1)$

In Exercises 19–22, find the area and perimeter of the figure determined by the points.

19. $(-5, 3), (0, -1), (4, 4)$

20. $(-2, -2), (-2, 2), (2, 2), (2, -2)$

21. $(-3, -1), (-1, 3), (7, 3), (5, -1)$

22. $(-2, 1), (-2, 6), (4, 6), (4, 1)$

In Exercises 23–28, find the midpoint of the line segment with the given endpoints.

23. $-9.3, 10.6$ **24.** $-5, -17$

25. $(-1, 3), (5, 9)$

26. $(3, \sqrt{2}), (6, 2)$

27. $(-7/3, 3/4), (5/3, -9/4)$

28. $(5, -2), (-1, -4)$

In Exercises 29–34, draw a scatter plot of the data given in the table.

29. U.S. Aluminum Imports The total value y in billions of dollars of aluminum imported by the United States each year from 1997 to 2003 is given in the table. (*Source: U.S Census Bureau, Statistical Abstract of the United States, 2001, 2004–2005.)*

x	1997	1998	1999	2000	2001	2002	2003
y	5.6	6.0	6.3	6.9	6.4	6.6	7.2

30. U.S. Aluminum Exports The total value y in billions of dollars of aluminum exported by the United States each year from 1997 to 2003 is given in the table. (*Source: U.S. Census Bureau, Statistical Abstract of the United States, 2001, 2004–2005.*)

x	1997	1998	1999	2000	2001	2002	2003
y	3.8	3.6	3.6	3.8	3.3	2.9	2.9