Today's Objective

Read **words problems concerning population growth** and decipher the real-world meaning by identifying key given information using sentence stems. Write the equation of an **exponential population model** from given information.

- Success Criteria
  - Define an exponential population model
  - Relate real world scenarios to exponential functions.

Vocabulary: constant percentage rate, maximum sustainable population.

Constant Percentage Rate

Suppose that a population is changing at a **constant percentage rate** ***r***, where r is the percent rate of change expressed in decimal form. Then the population follows the pattern shown.

Time in years	Population
0	$P(0) = P = $ initial population
1	$P(1) = P_0 + P_0 r = P_0(1+r)$
2	$P(2) = P(1)\cdot(1+r) = P_0(1+r)^2$
3	$P(3) = P(2)\cdot(1+r) = P_0(1+r)^3$
$\vdots$	$\vdots$
t	$P(t) = P_0(1+r)^t$

Exponential Population Model

If a population P is changing at a constant percentage rate r each year, then $P(t) = P_0(1+r)^t$,
where P_0 is the initial population,
r is expressed as a decimal,and t is time in years.

Example **Finding Growth and Decay Rates**

1+ .021

Let $P(t) = 786{,}543 \cdot (1.021)^t$. Graph the function and decide whether the population model $P(t) = 786{,}543 \cdot (1.021)^t$ is an exponential growth function or exponential decay function.

The base, $b =$ 1.021.

The initial value, $a =$ 786,543.

The exponent is the variable t or time.

Find, r, the constant percent rate of growth for the population model. 2.1%

Example **Finding Growth and Decay Rates**

$P(t) = 786{,}543 \cdot (1.021)^t$ is an exponential growth function, because the base, b, is greater than one.

Since the base, b, of the function is $1.021 = (1 + r)$ we can solve for r. Solving the linear equation yields $r =$ 2.1%. Since the rate, $r > 0$, P is an exponential growth function with a constant growth rate of .21 = 2.1 %.

.021

Example **Finding Growth and Decay Rates**

$P(t) = 786{,}543 \cdot (1.021)^t$ is an exponential growth function,

because the base, b, is ___________ then one.

$t = 10$ years $\Rightarrow P(10) = 786{,}543(1.021)^{10}$

LO: If we assume this is a finance problem then the real world meaning is given as follows. If a bank account is open with $786,543 and I don't withdraw any money but just let it grow then each year the account balance, or amount of money in the account, will increases or goes up by 2.1 percent of the previous year. If I am very frugal and don't take out any money then at the end of the tenth year my account will contain $968,233.

Example **Finding Growth and Decay Rates**

$P(t) = 786{,}543 \cdot (1.021)^t$ is an exponential growth function, because the base, b, is ___________ then one.

LO: If we assume this is a biology problem then the real world meaning is given as follows. If at the start of an experiment a biologist counts 786,543 bacteria in a petri dish and none of them die but just keep growing then after each hour the population, or amount of bacteria in the dish, will increase or go up by 2.1 percent of the previous hour. If I assume that no bacteria die then after 3 hours and 20 minutes the biologist will know the petri dish contains

842,963 bacteria.

Determining an Exponential Function

Determine the exponential function with initial value=10, increasing at a rate of 5% per year.

Because $P_0 = \underline{\quad 10 \quad}$ and

$r = \underline{\quad 5 \quad}\% = \underline{\quad .05 \quad}$,

the function is $P(t) = P_0(1+r)^t$

$P(t) = \underline{\quad 10 \quad}(1+\underline{\quad .05 \quad})^t$ or

$P(t) = \underline{\quad 10 \quad}(\underline{\quad 1.05 \quad})^t$.

WP: Exponential Functions

1. Write an exponential function to model the situation. Then find the value of the function after 5 years to the nearest whole number. A population of 250 animals that increases at an annual rate of 19%.

[A] $f(x) = 250(1.19)^x$; 597 ~~[B]~~ $f(x) = 250(0.81)^x$; 1013

~~[C]~~ $f(x) = 250(0.81)^x$; 87 [D] $f(x) = 250(1.19)^x$; 1488

$$f(5) = 250(1.19)^5$$

LO: We know that the initial value corresponds to a, so a = 250. Since the rate of growth is 19%, the base will be equal to 1.19. This means our equation will be $250(1.19)^x$. Since time, x = 5, we can evaluate the function for x = 5. This give us a population value of 597 after 5 years.

3

WP: Exponential Functions

Write an exponential function to model the situation. Tell what each variable represents.

2. A price of $135 increases 3% each month.

[A] $p = 135(1.03)^x$; x is the total price, and p is the number of months

[B] $p = 135(1.03)x$; x is the total price, and p is the number of months

[C] $p = 135(1.03)x$; p is the total price, and x is the number of months

[D] $p = 135(1.03)^x$; p is the total price, and x is the number of months

LO: We know that the initial value corresponds to a, so a = 135. Since the rate of growth is 3%, the r will be equal to .03. This means our equation will be $135(1.03)^x$.

3

WP: Exponential Functions

Write an exponential function to model the situation. Tell what each variable represents.

3. A price of \$115 increases 5% each month.

$f(x) = 115(1.05)^x$

LO: We know that the initial value corresponds to a, so a = 115. Since the rate of growth is 5%, the r will be equal to .05. This means our equation will be ________. Since time, x = _______, we can ____________ the function for x = _______. This give us a value of __________.

3

WP: Exponential Functions

Write an exponential function to model the situation. Tell what each variable represents.

4. Write an exponential function to model the situation. Then predict the value of the function after 5 years (to the nearest whole number). A population of 470 animals that decreases at an annual rate of 19%.

$$f(x) = 470(0.81)^x$$

$$f(5) = 163.87 \Rightarrow 164$$

LO: We know that the initial value corresponds to a, so a = 470. Since the rate of decay is 19%, the r will be equal to -.19. This means our equation will be see above. Since time, x = 5, we can evaluate the function for x = 5. This give us a value of ________.

3

Example **Modeling Bacteria Growth**

Suppose a culture of 200 bacteria is put into a petri dish and the culture doubles every hour. Predict when the number of bacteria will be 350,000.

Because $P_0 =$ 200 and $r =$ 100 % $=$ 1,

the function is $P(t) = P_0(1+r)^t =$ 200 $(1+$ 1 $)^t$.

Therefore I must solve the equation

$\underbrace{P(t)}_{y_1} = 200(2)^t$.

Solve this graphically by finding the

intersection of $P(t)$ and the horizontal line

$\underbrace{y = 350000}_{y_2}$.

10 hrs, 46 min

$x = 10.77$

Example **Modeling Bacteria Growth**

Suppose a culture of 200 bacteria is put into a petri dish and the culture doubles every hour. Predict when the number of bacteria will be 350,000.

LO: If at the start of an experiment a biologist counts __200__ bacteria in a petri dish and none of them die but just keep growing then after each hour the population, or amount of bacteria in the dish, will increase or go up by __100__ percent of the previous hour. If I assume that no bacteria die then after __10__ hours and __47__ minutes the culture will contain 350,000 bacteria.