Today's Objectives

- Orally characterize the unique graphical, numerical, and algebraic **properties of exponential functions**. Synthesize **the equation of an exponential function** from numeric data using guided notes samples.
- Success Criteria
  - Define Exponential Function
  - Differentiate between exponential decay and exponential growth
  - Describe transformations of exponential functions
- Vocabulary: exponential, base, growth factor, decay factor

Exponential Functions

Let a and b be real number constants.

An **exponential function** in x is a function that can be written in the form $f(x) = a \cdot b^x$, where a is nonzero, b is positive, and $b \neq 1$.

y=mx+b

The constant a is the **initial value** of f (the value at $x = 0$), and b is the **base**.

The domain/input of exponential functions are exponents.

The range/output is the base, *b*, multiplied by itself ***[insert exponent here]*** times and vertically stretched or shrunk by a factor of *a*.

Finding an Exponential Function from its Table of Values

Determine formulas for the exponential function g and h whose values are given in the table below.

x	$g(x)$		$h(x)$	
-2	4/9		128	
		$\times 3$		$\times \frac{1}{4}$
-1	4/3		32	
		$\times 3$		$\times \frac{1}{4}$
0	4		8	
		$\times 3$		$\times \frac{1}{4}$
1	12		2	
		$\times 3$		$\times \frac{1}{4}$
2	36		1/2	

$g(x) = 4 * 3^x$ $h(x) = 8 * \frac{1}{4}^x$

Because g is exponential, $g(x) = a \cdot b^x$.

Because $g(0) = 4$, $a = 4$.
Because $g(1) = 4 \cdot b^1 = 12$, the base $b = 3$. So, $g(x) = 4 \cdot 3^x$.

x	$g(x)$
-2	$4/9$
-1	$4/3$
0	4
1	12
2	36

$\times 3$
$\times 3$
$\times 3$
$\times 3$

Because h is exponential, $h(x) = a \cdot b^x$.

Because $h(0) = 8$, $a = 8$.

Because $h(1) = 8 \cdot b^1 = 2$, the base $b = 1/4$. So, $h(x) = 8 \cdot \left(\frac{1}{4}\right)^x$.

x	$h(x)$
-2	128
-1	32
0	8
1	2
2	1/2

$\times \frac{1}{4}$

$\times \frac{1}{4}$

$\times \frac{1}{4}$

$\times \frac{1}{4}$

Exponential Growth and Decay

For any exponential function $f(x) = a \cdot b^x$ and any real number x, $f(x+1) = b \cdot f(x)$.

If $a > 0$ and $b > 1$, the function f is increasing and is an **exponential growth function**. The base b is its **growth factor**.

If $a > 0$ and $b < 1$, the function f is decreasing and is an **exponential decay function**. The base b is its **decay factor**.

Example **Transforming Exponential Functions**

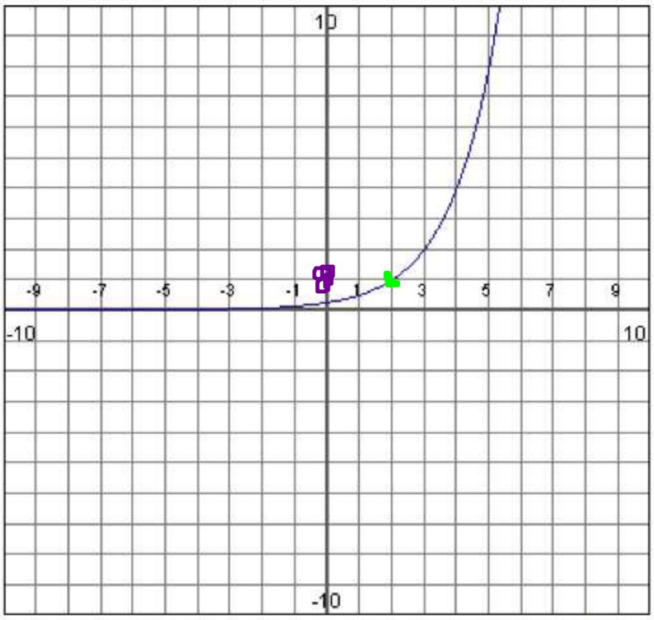
Describe how to transform the graph of $f(x) = 2^x$ into the graph of $g(x) = 2^{x-2}$.

The graph shifts two units right

Example **Transforming Exponential Functions**

Describe how to transform the graph of $f(x) = 2^x$ into the graph of $g(x) = 2^{x-2}$.

The graph of $g(x) = 2^{x-2}$ is obtained by translating the graph of $f(x) = 2^x$ by 2 units to the right.



Reflections: Transforming Exponential Functions

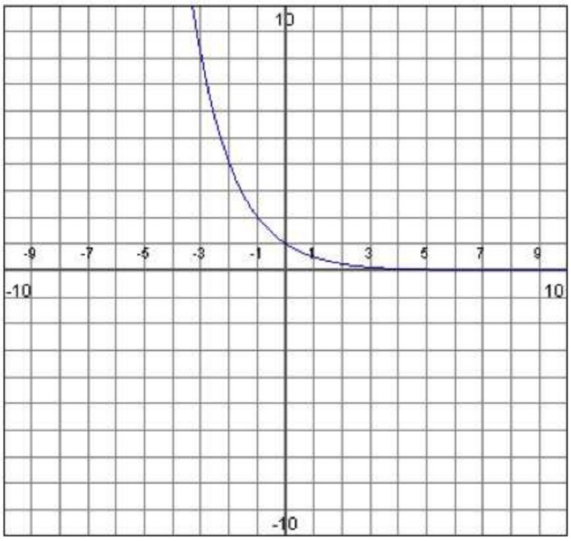
Describe how to transform the graph of $f(x) = 2^x$ into the graph of $g(x) = 2^{-x}$.

graph Reflects, y. axis

Reflections: Transforming Exponential Functions

Describe how to transform the graph of $f(x) = 2^x$ into the graph of $g(x) = 2^{-x}$.

Since the transformation is applied to x it will effect a horizontal property of the graph, therefore the graph of $g(x) = 2^{-x}$ is obtained by making a horizontal reflection of the graph.
So flip $f(x) = 2^x$ across the y-axis to produce $g(x) = 2^{-x}$.



Today's Objective

- Assess **the relationship between the natural base and logistic functions** and use a limit statement sentence frame to write a mathematical analysis **of the natural base**.
- Success Criteria
  - Define e
  - Explore properties of the exponential function with base e
  - Define a logistic growth function

- Vocabulary: e, natural base, logistic growth function, limit to growth

The Natural Base *e*

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

Turn and Talk:

- When have we used limits? In your experience, what do limits tell us?
- Think about polynomial and rational functions what type of limit is indicated when *x* approaches infinity?
- Find the limit numerically, and make a conjecture about the value of Euler's constant also called the natural base *e*.

Report to me:

- **Each group must state there answer to the teacher.**

Exponential Functions and the Base e

Any exponential function $f(x) = a \cdot b^x$ can be rewritten as $f(x) = a \cdot e^{kx}$, for any appropriately chosen real number constant k.

If $a > 0$ and $k > 0$, $f(x) = a \cdot e^{kx}$ is an exponential growth function.

If $a > 0$ and $k < 0$, $f(x) = a \cdot e^{kx}$ is an exponential decay function.

Turn and Talk:
What is the relationship between *b, k*, and *e*?

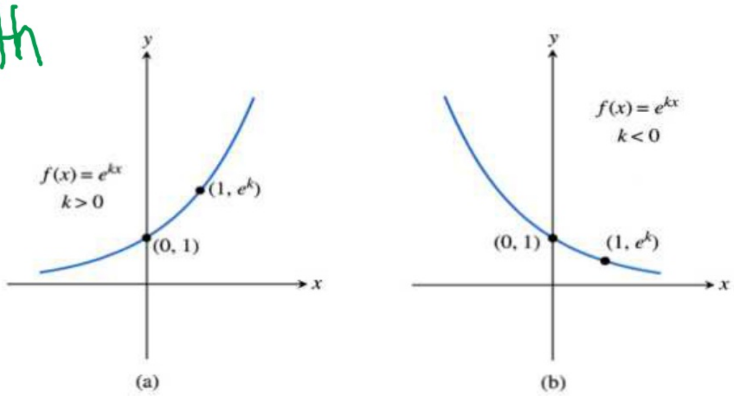
Justify your response using the properties of exponents.

Report to me:

- **Each group must state there answer to the teacher.**

Exponential Functions and the Base e

growth decay



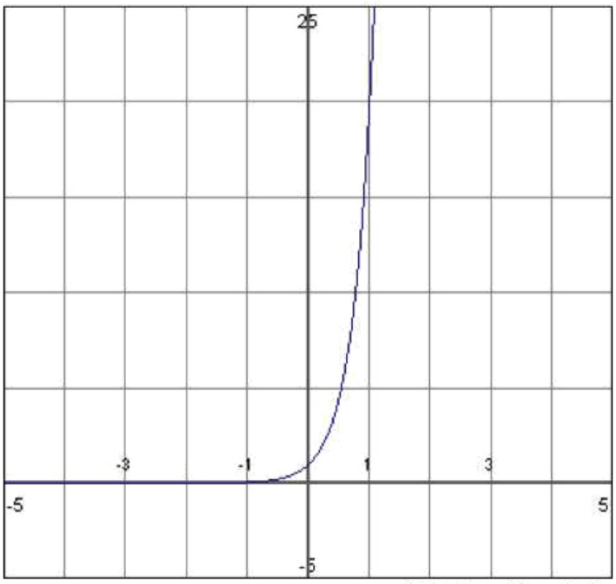
Turn and Talk:
- Classify these functions.
- What will they always have in common?
- Use the properties of exponents to explain why?

Report to me:
- **Each group must state there answer to the teacher.**

Example **Transforming Exponential Functions**

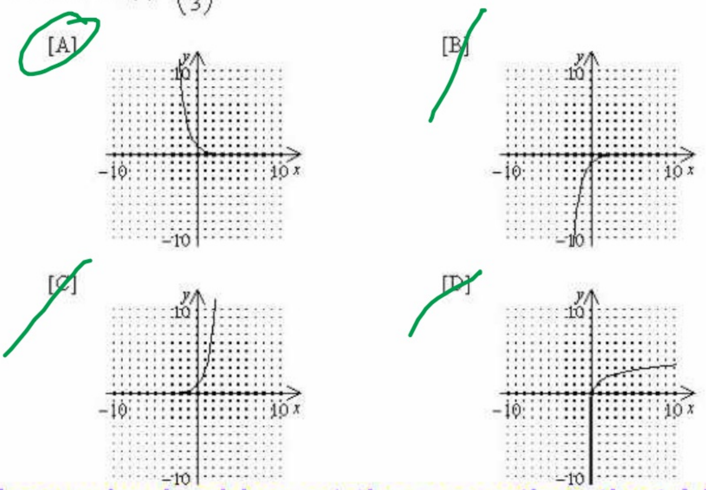
Describe how to transform the graph of $f(x)=e^x$ into the graph of $g(x)=e^{3x}$.

Since the transformation is applied to x it will effect a horizontal property of the graph, therefore the graph of $g(x)=e^{3x}$ is obtained by making a horizontal Shrink of the graph.



AM: Graph exponential functions

1. Graph: $f(x) = \left(\frac{1}{3}\right)^x$



LO: I know by looking at the equation that I have an *a* = __1__ > __0__, and a *b* = __1/3__ < __1__. This means I have an example of exponential __decay__ with a y-intercept of __(0, 1)__. Therefore, my answer is __A__.

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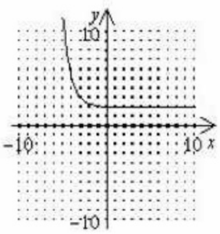
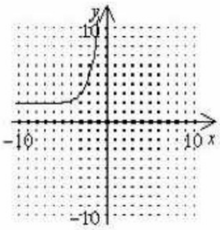
Graph Exponential Functions

1. Graph: $f(x) = \left(\frac{1}{3}\right)^{x-3} - 2$

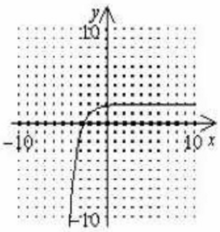
~~[A]~~



~~[B]~~



[C]



[D] none of these

LO: I know by looking at the equation that *a* = __1__ > __0__, and *b* = __1/3__ < __1__ value. This means I have an example of exponential __decay__ with a y-intercept of __0, 1__. I also have a __horizontal__ shift of __3__ units __right__ and a __vertical__ shift of __2__ units __down__. Therefore, my answer is __D__.

Report to me:

- **Each group must state the answer to the teacher.**
- **Pick up your AM Practice on Objectives #1 and #2**

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