AM Objective 12: Solve Inequalities with Rational Expressions

1. Solve: $\frac{(x-5)(x+3)}{x-3} \le 0$

[A] $x \le -3$ or $3 < x \le 5$ [B] $3 \le x \le -5$

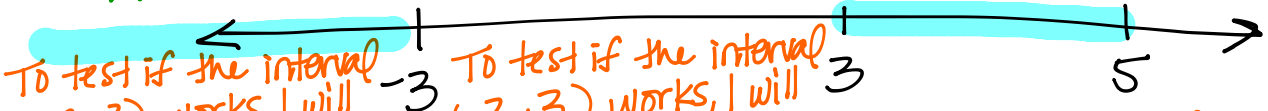
[C] $x \le -3$ or $x \ge -5$ [D] $x \ge 5$ or $-3 \le x < 3$

Since $x=3$ is undefined, we need to check what happens on each side of 3.

We also know, 5 and -3 will cause the left side of the inequality to equal zero. This means I should check -3, 3, & 5.

$x \le -3$ OR $3 < x \le 5$

* 3 can't be equal since 3 is not in domain.



To test if the interval $(-\infty, -3)$ works, I will evaluate for $x=-4$

$\frac{(-4-5)(-4+3)}{-4-3} \le 0$

$\frac{-9}{7} \le 0$ Yes!

To test if the interval $(-3, 3)$ works, I will evaluate for $x=0$

$\frac{(0-5)(0+3)}{0-3} \le 0$

$5 \le 0$ NO!

To test if the interval $(3, 5)$ works, I will evaluate for $x=4$

$\frac{(4-5)(4+3)}{4-3}$

$-7 \le 0$ yes!

To test if the interval $(5, \infty)$ works, I will evaluate for $x=6$

$\frac{(6-5)(6+3)}{(6-3)} \le 0$

$3 \le 0$ NO!

2. Identify the solution set of the inequality: $\frac{7x+1}{x-1} \le 9$

[A] $x \ge 5$ [B] $1 < x \le 5$ [C] $x < 1$ or $x \ge 5$ [D] $1 \le x \le 5$

Get zero on one side

$\frac{7x+1}{x-1} \le 9$

$-9 \quad -9$

$\frac{7x+1}{x-1} - 9 \le 0$

$\frac{7x+1}{x-1} - \frac{9(x-1)}{(x-1)} \le 0$

get LCD

$\frac{7x+1-9(x-1)}{(x-1)} \le 0$

$\frac{7x+1-9x+9}{(x-1)} \le 0$

$\frac{-2x+10}{x-1} \le 0$

$\frac{-2x+10}{x-1} \le 0$ ← We are looking for where the function is negative or zero.

The function equals zero at $x=5$

The function is undefined at $x=1$

cannot be equal since $x=1$ is undefined

$x < 1$ OR $x \ge 5$

Test $x=0$: $\frac{-2(0)+10}{0-1}$, $\frac{10}{-1}$, $-10 = \ominus$

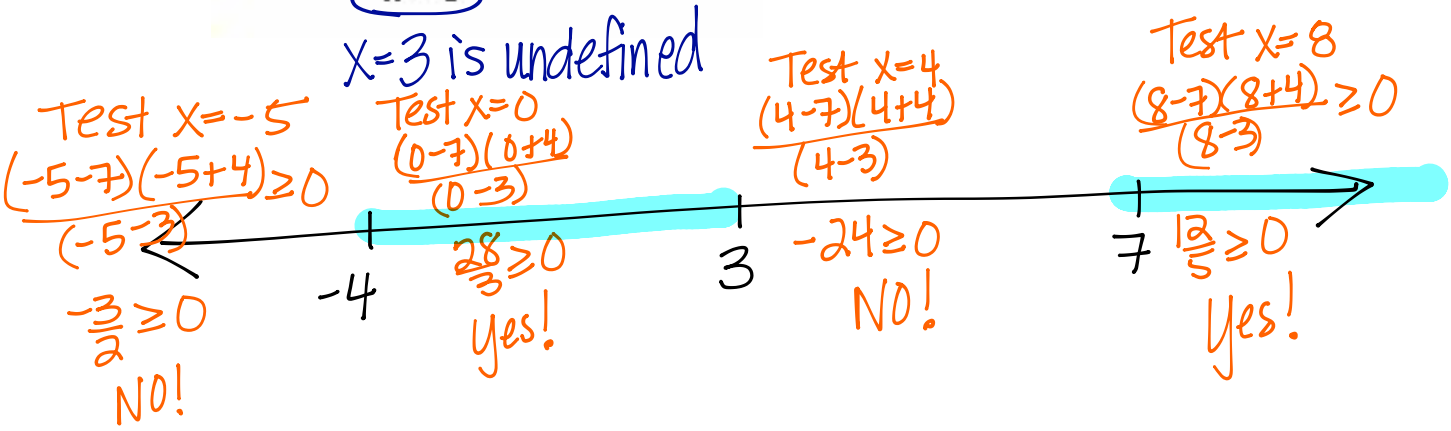
Test $x=2$: $\frac{-2(2)+10}{2-1}$, $\frac{6}{1}$, $6 = \oplus$

Test $x=6$: $\frac{-2(6)+10}{6-1}$, $\frac{-2}{5} = \ominus$

$x=7$ and $x=-4$ will cause the left side to equal 0.

3. $\dfrac{(x-7)(x+4)}{x-3} \geq 0$

$x=3$ is undefined



$-4 \leq x < 3$ OR $x \geq 7$

↑ Can't equal 3, since $x=3$ is undefined.

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4. $\dfrac{x+1}{x-1} \leq 3$

Get zero on one side

$\dfrac{x+1}{x-1} - 3 \leq 3 - 3$

$\dfrac{x+1}{x-1} - \dfrac{3(x-1)}{(x-1)} \leq 0$ ← LCD

$\dfrac{x+1}{x-1} - \dfrac{3(x-1)}{x-1} \leq 0$

$\dfrac{x+1-3(x-1)}{(x-1)} \leq 0$

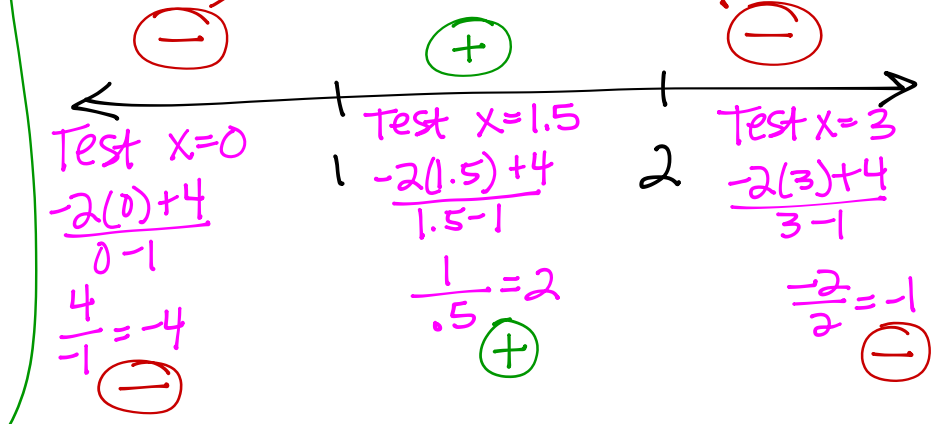
$\dfrac{x+1-3x+3}{x-1} \leq 0$

$\boxed{\dfrac{-2x+4}{x-1} \leq 0}$

$\dfrac{-2x+4}{x-1} \leq 0$ ← We are looking for where the function is negative or zero.

The function will equal zero at $x=2$

The function will be undefined at $x=1$

Cannot be equal since $x=1$ is undefined

$x < 1$ OR $x \geq 2$