Name *Key* Date______________

Precalculus[redacted] – Polynomial Zeros and Rational Functions

1. Divide the polynomial $p(x)=1+5x^6+2x^4+10x^5+2x^2+5x^3+3x$ by $d(x)=x^2+2x$.

Divisor	$d(x) = x^2+2x+0$
Dividend	$p(x) = 5x^6+10x^5+2x^4+5x^3+2x^2+3x+1$
Quotient	$q(x) = 5x^4+2x^2+x$
Remainder	$r(x) = 3x+1$
Fraction Form	$\frac{p(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$
Polynomial Form	$p(x) = q(x)\cdot d(x) + r(x)$

$$\begin{array}{r} 5x^4 \quad\quad + 2x^2 + x \\ x^2+2x+0 \overline{)\,5x^6+10x^5+2x^4+5x^3+2x^2+3x+1} \\ -(5x^6+10x^5+0x^4) \\ \hline 2x^4+5x^3+2x^2 \\ -(2x^4+4x^3+0x^2) \\ \hline x^3+2x^2+3x+1 \\ -(x^3+2x^2+0x) \\ \hline 3x+1 \end{array}$$

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2. Given the function $f(x) = 7x^4 + 15x^3 - 12x^2 - 30x - 4$

a) List all of the possible rational zeros of $f(x)$.

p	q	p/q
$\pm 1, 2, 3, 4$	$\pm 1, \pm 7$	$\pm 1, \pm 2, \pm 3, \pm 4$, $\pm \frac{1}{7}, \pm \frac{2}{7}, \pm \frac{3}{7}, \pm \frac{4}{7}$

$f(-\frac{1}{7}) = 0 \quad f(-2) = 0$

b) Use synthetic division to find the actual zeros of $f(x)$ and complete the table below.

$$\begin{array}{r|rrrrr} -2 & 7 & 15 & -12 & -30 & -4 \\ & \downarrow & -14 & -2 & +28 & +4 \\ \hline & 7 & 1 & -14 & -2 & 0 \end{array}$$

$f(x) = (x+2)(7x^3 + x^2 - 14x - 2)$

$$\begin{array}{r|rrrr} -\frac{1}{7} & 7 & 1 & -14 & -2 \\ & \downarrow & -1 & 0 & +2 \\ \hline & 7 & 0 & -14 & 0 \end{array}$$

$f(x) = (x+2)(x+\frac{1}{7})(7x^2 - 14) = 7(x+2)(x+\frac{1}{7})(x^2 - 2)$

$x^2 - 2 = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$

$f(x) = 7(x+2)(x+\frac{1}{7})(x+\sqrt{2})(x-\sqrt{2})$
OR
$= (7x+1)(x+2)(x+\sqrt{2})(x-\sqrt{2})$

Zero or Root	x-intercept	Linear Factor	Multiplicity
$x = -2$	$(-2, 0)$	$(x+2)$	1
$x = -1/7$	$(-1/7, 0)$	$(x + 1/7)$	1
$x = \sqrt{2}$	$(\sqrt{2}, 0)$	$(x - \sqrt{2})$	1
$x = -\sqrt{2}$	$(-\sqrt{2}, 0)$	$(x + \sqrt{2})$	1
The factored form of $f(x)$ is …	(see factored form above)		

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Precalculus [redacted] – Polynomial Zeros and Rational Functions

3. Use polynomial theorems to make predictions about the zeros of the given polynomial $p(x) = 2x^4 + x^3 + 7x^2 + 5x - 1$ DO NOT FIND THE ZEROS.

a) Use the Intermediate Value Theorem (IVT) to conclude if possible whether or not the function $p(x)$ has a zero on the interval **[-2,2]**. Justify.

$p(-2) = 41 > 0$

$p(2) = 77 > 0$

Since $p(-2) = 41 > 0$ and $p(2) = 77 > 0$, the endpoints of the tested interval are both positive, that is have the same sign and therefore the IVT can not be used to make a conclusion about the zeros of p on $[-2,2]$.

b) Use $\mathbf{x = 5}$ with a bounds tests to give an upper or lower bound for the zeros of $p(x)$. Justify.

5	2	1	7	5	-1
	↓	10	55	310	1575
	2	11	62	315	1574

Since performing synthetic division on $p(x)$ with $k = 5 > 0$ resulted in a quotient with all nonnegative coefficients, $k = 5$ is an upper bound for the real zeros of $p(x)$.

c) Use Descartes' Rule of Signs to make a prediction about the number of **positive** real zeros for $p(x)$. Justify.

We compared the signs of the coefficients of $p(x)$ to determine that $p(x)$ has 1 sign change in its coefficients, therefore $p(x)$ has exactly 1 positive real root.