

AM Objective #19: Find inverses of functions

1. Find the inverse of the function: $f(x) = \{(-6, -4), (-4, -6), (7, 0)\}$

(A) $f^{-1} = \{(-4, -6), (-6, -4), (0, 7)\}$

(B) $f^{-1} = \{(-4, -6), (-6, 7), (0, 7)\}$

(C) $f^{-1} = \{(-4, -6), (-6, 0), (0, 7)\}$ (D) $f^{-1} = \{(-6, -4), (-4, -6), (7, 0)\}$

Inverses are a switch of domain and range, $x \leftrightarrow y$.

$f(x) = (-6, -4) \quad (-4, -6) \quad (7, 0)$
 $\downarrow \quad \quad \downarrow \quad \quad \downarrow$
 $f^{-1}(x) = (-4, -6) \quad (-6, -4) \quad (0, 7)$

2. Determine the equation for the inverse function of $y = (x - 8)^3 - 9$.

(A) $y = \sqrt[3]{x+1}$ (B) $y = \sqrt[3]{x+9} + 8$ (C) $y = \sqrt[3]{x} + 17$ (D) none of these

① Switch x and y

$$y = (x-8)^3 - 9$$

$$\downarrow$$

$$x = (y-8)^3 - 9$$

② Solve for y

$$x = (y-8)^3 - 9$$

$$\sqrt[3]{x+9} = \sqrt[3]{(y-8)^3}$$

$$\sqrt[3]{x+9} = y-8$$

$$\sqrt[3]{x+9} + 8 = y$$

The 8 must stay on the outside!

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3. Find the inverse, $f^{-1}(x)$, of the function $f(x) = \frac{-1+x}{2-x}$, if it exists.

[A] $\frac{1-x}{-1+2x}$

[B] $\frac{-2x-1}{-x-1}$

[C] $\frac{-x+2}{x-1}$

[D] $f^{-1}(x)$ does not exist.

① Switch x and y

$$y = \frac{-1+x}{2-x}$$

↓

$$x = \frac{-1+y}{2-y}$$

② Solve for y

$$(2-y)x = \frac{-1+y}{2-y} \cdot (2-y)$$

$$2x - xy = -1 + y$$

Get all terms with y on one side

$$2x - xy = -1 + y$$

$$-2x - y \quad -2x - y$$

$$-xy - y = -2x - 1$$

↓ factor out y

$$y(-x-1) = -2x-1$$

$$\frac{y(-x-1)}{(-x-1)} = \frac{-2x-1}{-x-1}$$

$$y = \frac{-2x-1}{-x-1}$$

4. Find the inverse, $f^{-1}(x)$, of the function $f(x) = \frac{3x}{x+4}$, if it exists.

[A] $f^{-1}(x) = \frac{x+4}{3x}$

[B] $f^{-1}(x) = \frac{4x}{-x+3}$

[C] $f^{-1}(x) = \frac{3x}{-1-4x}$

[D] $f^{-1}(x)$ does not exist.

① Switch x and y

$$y = \frac{3x}{x+4}$$

↓

$$(y+4)x = \frac{3y}{y+4} \cdot (y+4)$$

$$xy + 4x = 3y$$

Get all terms with y on one side

$$xy + 4x = 3y$$

$$-3y - 4x \quad -3y - 4x$$

$$xy - 3y = -4x$$

$$y(x-3) = -4x$$

$$y \frac{(x-3)}{(x-3)} = \frac{-4x}{(x-3)}$$

$$y = \frac{-4x}{x-3}$$

Rewrite as $f^{-1}(x)$

$$f^{-1}(x) = \frac{-4x}{x-3}$$

$$+ \frac{(4x)}{(-x+3)}$$