# Today's Objectives

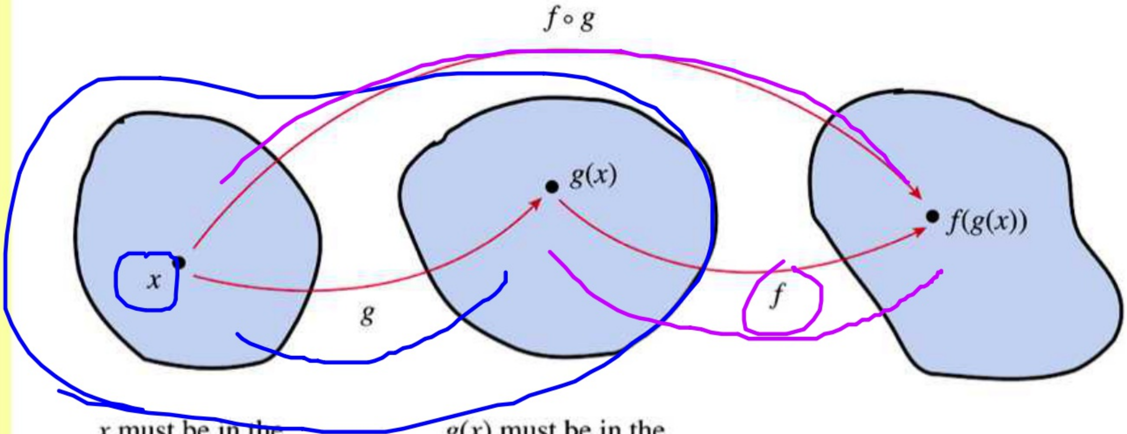
- **Produce new functions by composing existing functions and evaluate for given values** after listening to a step-by-by explanation with key words.
- Success Criteria
  - Define composition and notation
  - Assess functions for compatible domains and ranges
- Vocabulary: composition, compatible

# Composition of Functions

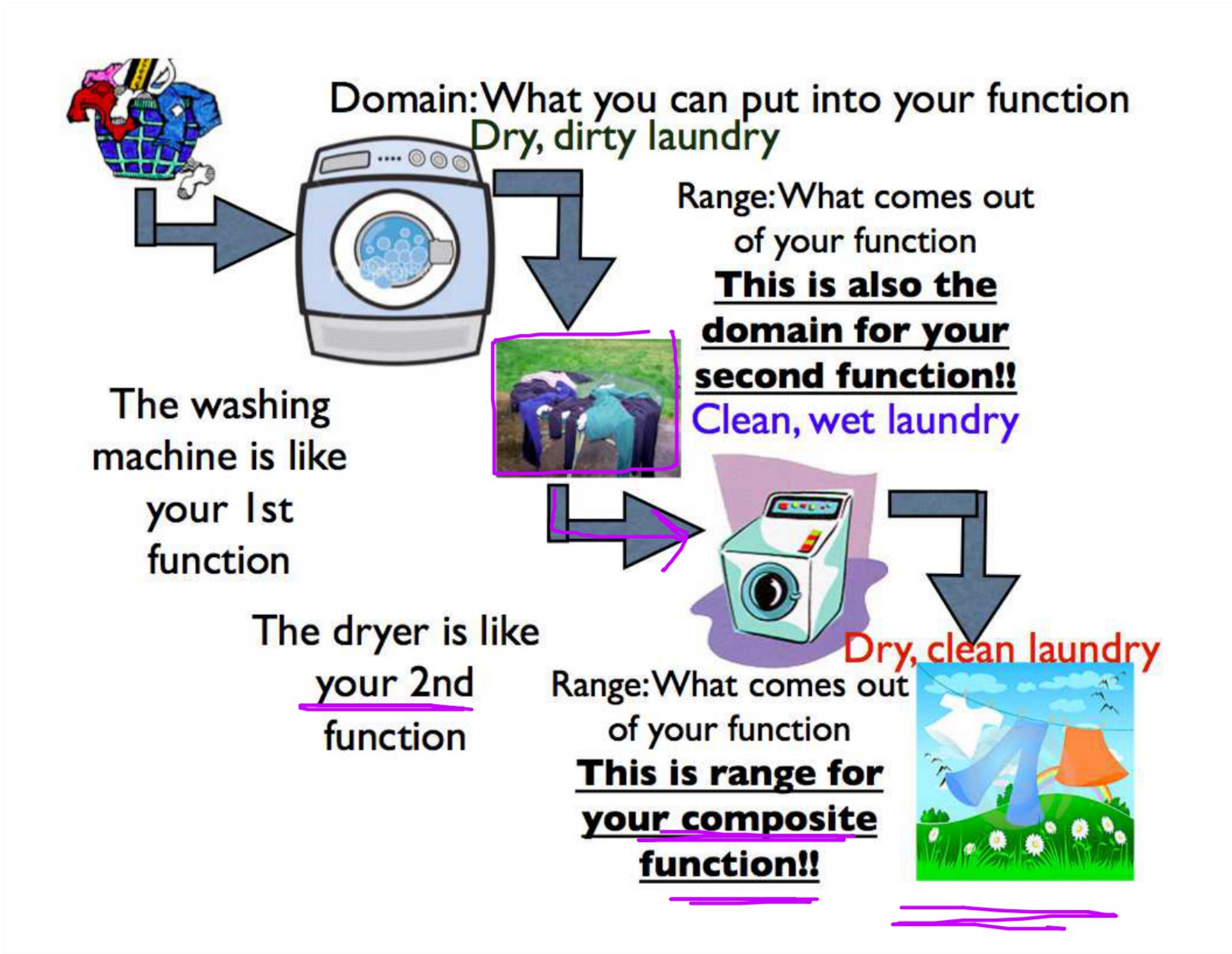
Let $f$ and $g$ be two functions such that the domain of $f$ intersects the range of $g$. The composition $f$ of $g$, denoted $\boxed{f \circ g}$, is defined by the rule $(f \circ g)(x) = f(g(x))$.

The domain of $f \circ g$ consists of all $x$-values in the domain of $g$ that map to $g(x)$-values in the domain of $f$.

# Composition of Functions




$x$ must be in the domain of $g$ and $g(x)$ must be in the domain of $f$


Domain: What you can put into your function
Dry, dirty laundry
Range: What comes out of your function
**This is also the domain for your second function!!**
Clean, wet laundry
The washing machine is like your 1st function
The dryer is like your 2nd function
Dry, clean laundry
Range: What comes out of your function
**This is range for your composite function!!**

# Example Composing Functions

Let $f(x) = 2^x$ and $g(x) = \sqrt{x+1}$. Find

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

Let $f(x) = 2^x$ and $g(x) = \sqrt{x+1}$. Find

(a) $\boxed{(f \circ g)(x)} = \boxed{f(g(x))}$

$f(\sqrt{x+1}) = 2^{\sqrt{x+1}}$

Let $f(x) = 2^x$ and $g(x) = \sqrt{x+1}$. Find

(a) $\left(g \circ f\right)\left(x\right) = g(f(x))$

$g(2^x) = \sqrt{2^x + 1}$

# Example **Composing Functions**

Let $f(x) = -x^2 + 4$ and $g(x) = \sqrt{x}$. Find

(a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x})$

$= -(\sqrt{x})^2 + 4$

$= -x + 4$

(b) $(g \circ f)(x)$

$g(f(x)) = g(-x^2+4)$

$= \sqrt{-x^2+4}$

# AM: Find compositions of 2 functions

**1.** If $f(x) = x^4$ and $g(x) = 1-2x^2$, find $g(f(x))$.

[A] $1-2x^8$ [B] $\dfrac{x^4}{1-2x^2}$ [C] $(1-2x^2)^4$ [D] $x^4-2x^6$

$$g(x^4) = 1-2(x^4)^2$$

$$= 1-2x^8$$

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## AM: Find compositions of 2 functions

**2.** Given $f(x) = \dfrac{x+5}{x}$ and $g(x) = x^2 + 4$, find $(g \circ f)(6)$.

[A] $\dfrac{520}{121}$ [B] $\dfrac{9}{8}$ [C] $\dfrac{265}{36}$ [D] $\dfrac{35}{6}$

$$g(\underbrace{F(6)}) = g\left(\frac{11}{6}\right) = \left(\frac{11}{6}\right)^2 + 4$$

$$F(6) = \left(\frac{6+5}{6}\right) = \frac{11}{6}$$

$$= \frac{121}{36} + 4$$

$$= \frac{265}{36}$$

# AM: Find Composition of 2 functions

**3.** Given $f(x) = -2x^2$, $g(x) = -3x+7$, and $h(x) = \sqrt{x}$, find $[(f+g)\circ h](x)$.

[A] $-2x-3\sqrt{x}+7$ [B] $-2x^2-3\sqrt{x}+7$ [C] $6x+\sqrt{x}+7$ [D] $-2\sqrt{x}-3x+7$

$[\underbrace{(f+g)}\circ h](x)$

1) $f(x)+g(x)$

$\underbrace{-2x^2 + -3x+7}$

$B(x) = -2x^2 - 3x + 7$

$(B\circ h)(x)$

$B(\sqrt{x})$

$-2(\cancel{\sqrt{x}})^{\cancel{2}} - 3(\sqrt{x}) + 7$

$\boxed{-2x - 3\sqrt{x} + 7}$

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## Today's Objectives

- Algebraically verify in writing **that two functions are inverses** and produce **inverse functions** using a step-by-step process and the algebraic definition of an inverse.
- Success Criteria:
  - Understand inverses in terms of dependency
  - Determine if a function is one-to-one
  - Identify graphical properties of inverses
- Vocabulary: inverse

## Inverse Relation

The ordered pair (a,b) is in a relation if and only if the pair (b,a) is in the inverse relation.

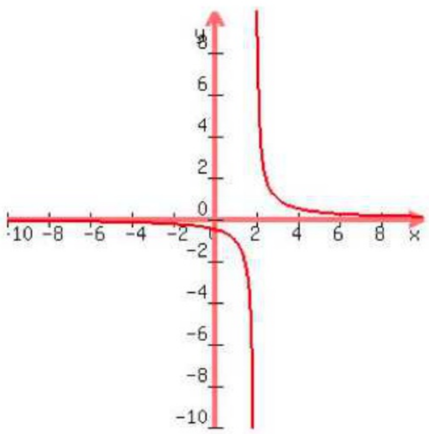
An inverse relationship represents a change in dependency, meaning that we are changing our dependent and independent variable. This means that x and y change places.

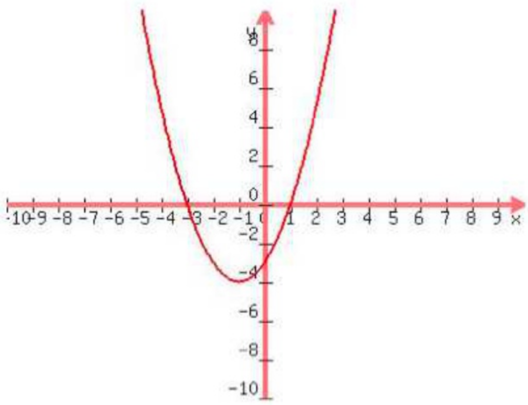
- i.e. (3, -8) becomes (-8, 3)
- Real world example

## Horizontal Line Test

The inverse of a relation is a function if and only if each horizontal line intersects the graph of the original relation in at most one point.

Do these functions pass?



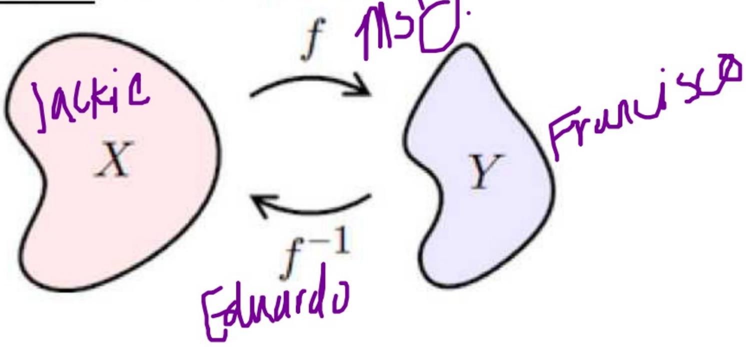
# Inverse Function

If $f$ is a one-to-one function with domain $D$ and range $R$, then the **inverse function of $f$**, denoted $f^{-1}$, is the function with domain $R$ and range $D$ defined by $f^{-1}(b) = a$ if and only if $f(a) = b$.

$f^{-1}$

## More about Inverses

- We write inverse functions as $f^{-1}$
- The domain of the original function is the range of the inverse
- The range of the original function is the domain of the inverse



## When can I have an inverse?

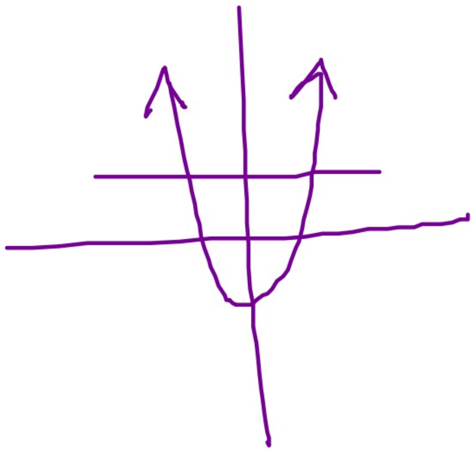
- There can only be an inverse when functions are one-to-one.
- One-to-one functions pass the vertical AND horizontal line test

- In relationships, a relationship is a one to one function when both people are only seeing one person.

# AM: Determine if functions are one-to-one

**1.** Which of the following is *not* a one-to-one function?

[A] $f(x) = x^2 - 2$ [B] $f(x) = \frac{1}{5}(x-2)$ [C] $f(x) = x - 2$ [D] $f(x) = 2x$



# AM: Determine if functions are one-to-one

**2.** Which of the following is a one-to-one function?

[A] $\{(9, -4), (8, -7), (5, 5), (8, 6)\}$

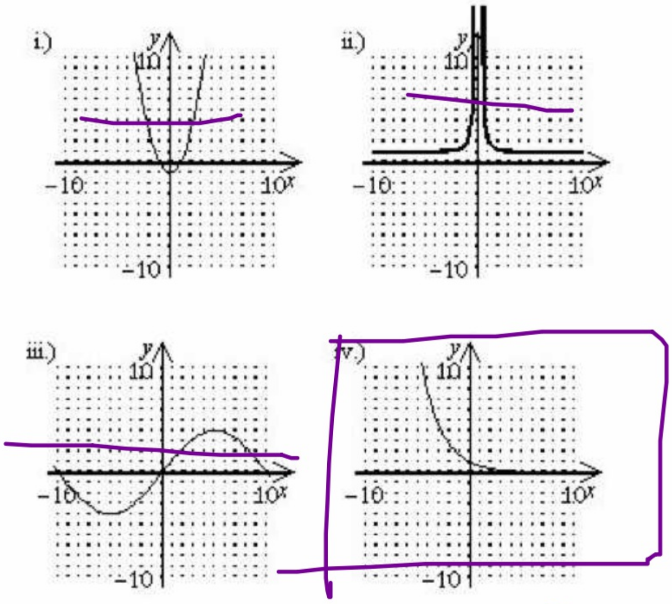
[B] $\{(9, -4), (8, 5), (5, 8), (1, 1)\}$

[C] $\{(9, -4), (8, -7), (9, -2), (-7, 8)\}$

[D] $\{(9, -4), (8, -7), (5, -2), (1, -4)\}$

# AM: Determine if functions are one-to-one

**3.** Determine which of the following are one-to-one functions:




[A] ii and iv only [B] i and iv only [C] iv only [D] i, ii, and iv only

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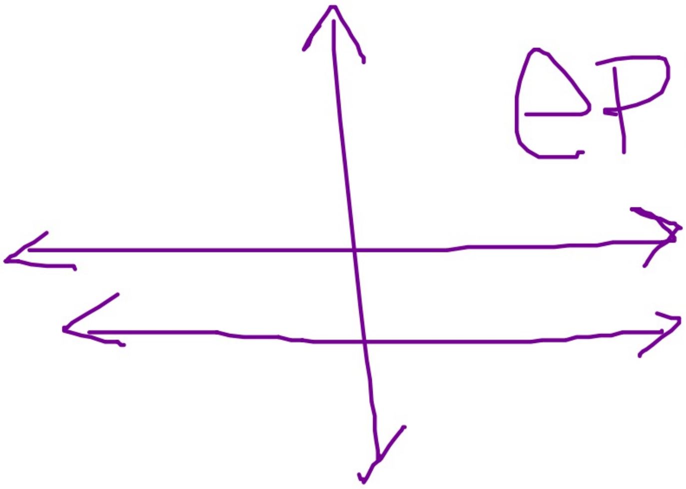
## AM: Determine if functions are one-to-one

**1.** Which of the following is *not* a one-to-one function?

[A] $f(x) = -2$ [B] $f(x) = \frac{1}{5}(x+2)$ [C] $f(x) = x+2$ [D] $f(x) = -2x$



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