How to Find an Inverse Function Algebraically

Given a formula for a function f, proceed as follows to find a formula for f^{-1}.

1. Determine that there is a function f^{-1} by checking that f is one-to-one. State any restrictions on the domain of f.

2. Switch x and y in the formula $y = f(x)$.

3. Solve for y to get the formula for $y = f^{-1}(x)$. State any restrictions on domain of f^{-1}.

Example Finding an Inverse Function Algebraically

Find an equation for $f^{-1}(x)$ if $f(x) = \dfrac{2x}{x-1}$.

$x = \dfrac{2y}{y-1}$ Switch the x and y

Solve for y:

$x(y-1) = 2y$ Multiply by $y-1$

$xy - x = 2y$ Distribute x

$xy - 2y = x$ Isolate the y terms

$y(x-2) = x$ Factor out y

$y = \dfrac{x}{x-2}$ Divide by $x-2$

Therefore $f^{-1}(x) = \dfrac{x}{x-2}$.

AM: Find inverses of relations

(−2, −6) (−6, −2) (−5, −1)

1. Find the inverse of the relation $f = \{(-6, -2), (-2, -6), (-1, -5)\}$.

[A] $\{(-2, -6), (-6, -2), (-5, -1)\}$

[B] $\{(-6, -2), (-2, -6), (-1, -5)\}$

[C] $\{(-2, -6), (-6, -5), (-5, -1)\}$

[D] $\{(-2, -6), (-6, -1), (-5, -1)\}$

AM: Find inverses of relations

2. Find the inverse of the relation $y = 2x + 4$.

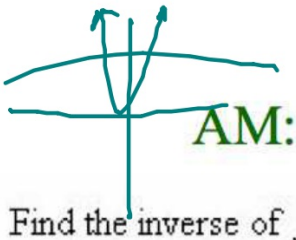
[A] $y = \frac{2x-4}{2}$ [B] $y = \frac{x+4}{2}$ [C] $y = 4x+2$ [D] $y = \frac{x-4}{2}$

1. one-to-one ✓
2. Switch x & y $x = 2y + 4$
3. Solve for y

$-4 \quad -4$

$\frac{x-4}{2} = \frac{2y}{2}$

$\frac{x-4}{2} = y$



AM: Find inverses of functions

$y = 17x^2$

1. Find the inverse of $f(x) = 17x^2$. Determine if the inverse is a function.

[A] $f^{-1}(x) = \pm\sqrt{\frac{x}{17}}$, $f^{-1}(x)$ is not a function.

[B] $f^{-1}(x) = \pm\frac{1}{17}\sqrt{x}$, $f^{-1}(x)$ is a function.

[C] $f^{-1}(x) = \pm\sqrt{\frac{x}{17}}$, $f^{-1}(x)$ is a function.

[D] $f^{-1}(x) = \pm\frac{1}{17}\sqrt{x}$, $f^{-1}(x)$ is not a function.

① one-to-one NO!

② Switch x & y

$\frac{x}{17} = \frac{17y^2}{17}$

③ Solve for y

$\pm\sqrt{\frac{x}{17}} = \sqrt{y^2}$

$\pm\sqrt{\frac{x}{17}} = y$ $f^{-1}(x)$

AM: Find inverses of functions

2. Determine the equation for the inverse function of $y = (x - 4)^3 + 9$.

[A] $y = \sqrt[3]{x-13}$ [B] $y = \sqrt[3]{x-9} + 4$ [C] $y = \sqrt[3]{x+4} - 9$ [D] $y = \sqrt[3]{x} - 5$

1. one-to-one ✓
2. switch x & y
3. Solve for y

$$x = (y-4)^3 + 9$$

$$-9 \qquad -9$$

$$\sqrt[3]{x-9} = \sqrt[3]{(y-4)^3}$$

$$\sqrt[3]{x-9} = y-4$$

$$+4 \qquad +4$$

$$\sqrt[3]{x-9} + 4 = y$$

AM: Find inverses of functions

3. Find the inverse, $f^{-1}(x)$, of the function $f(x) = \dfrac{-1+x}{2-x}$, if it exists.

[A] $\dfrac{1-x}{-1+2x}$ [B] $\dfrac{-2x-1}{-x-1}$ [C] $\dfrac{-x+2}{x-1}$ [D] $f^{-1}(x)$ does not exist.

1. one-to-one
2. Switch x & y
3. Solve for y

$(2-y)x = \dfrac{-1+y}{\cancel{2-y}} \cdot \cancel{(2-y)}$

$2x - xy = -1 + y$

$-2x \quad -y \quad -2x - y$

$-xy - y = -2x - 1$

$2x - xy = -1 + y$

* Now, get everything w/ a y on one side and everything else on the other side.

* Factor out y

$y\dfrac{(-x-1)}{(-x-1)} = \dfrac{-2x-1}{-x-1}$

$y = \dfrac{-2x-1}{-x-1} = \dfrac{-1(2x+1)}{-1(x+1)}$