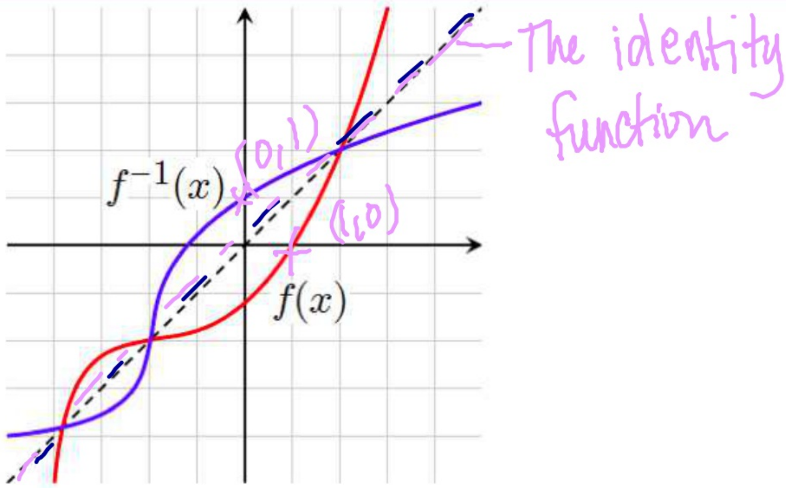
The Inverse Reflection Principle:

use to graphically "confirm" that functions are inverses

The points (a,b) and (b,a) in the coordinate plane are symmetric with respect to the line y=x. The points (a,b) and (b,a) are reflections of each other across the line y=x.



The Inverse Composition Rule:

Use to algebraically prove that functions are inverses of each other.

A function f is one-to-one with inverse function g if and only if

$f(g(x)) = x$ for every x in the domain of g, and

$g(f(x)) = x$ for every x in the domain of f.

composition

Example Verifying Inverse Functions

Show algebraically the $f(x) = x^3 + 2$ and $g(x) = \sqrt[3]{x-2}$ are inverse functions.

$f(g(x))$

$f\left(\sqrt[3]{x-2}\right)$

$\left(\sqrt[3]{x-2}\right)^3 + 2$

$x - 2 + 2$

x ✓

$g(f(x))$

$g(x^3+2)$

$\sqrt[3]{x^3+\cancel{2}-\cancel{2}}$

$\sqrt[3]{x^3}$

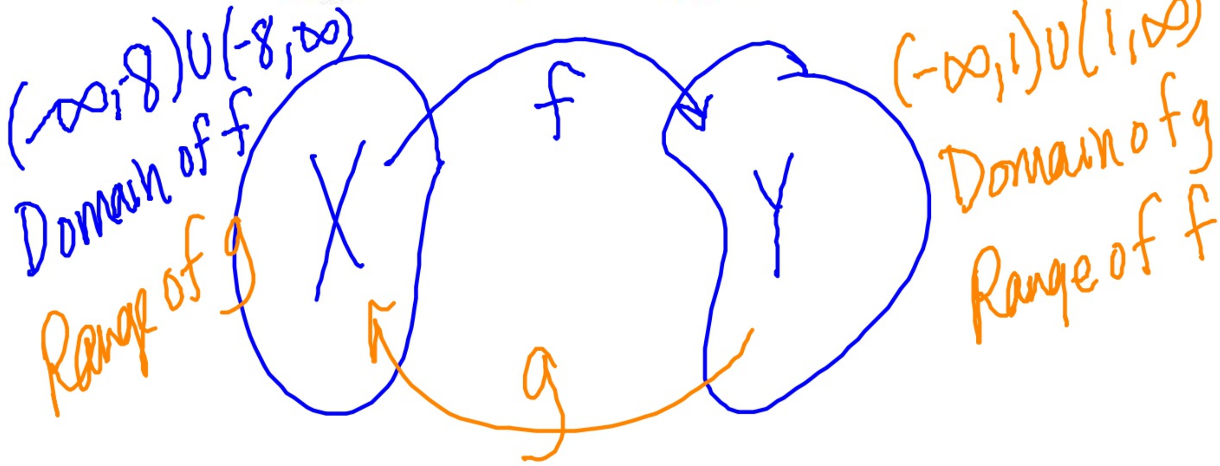
x ✓

Since $f(g(x)) = x$ and $g(f(x)) = x$, f and g are inverses.

Quiz #3 Practice

- Let $f(x)=\dfrac{x-7}{x+8}$ and $g(x)=\dfrac{-8x-7}{x-1}$, the functions *f(t)* and *g(t)* are inverses.

  a) Draw a function map that reveals the relationships between *f(t)* and *g(t)* and their respective domains and ranges in interval notation.



Quiz Practice #3

b. Confirm algebraically that *f* and *g* are inverses by showing that *f(g(x))* = *x* and *g(f(x))* = *x*.

$$f(x)=\frac{x-7}{x+8} \qquad g(x)=\frac{-8x-7}{x-1}$$

- Left side f(g(x))

$$f\left(\frac{-8x-7}{x-1}\right)=\frac{\left(\frac{-8x-7}{x-1}-7\right)\frac{x-1}{1}}{\left(\frac{-8x-7}{x-1}+8\right)\frac{x-1}{1}}$$

$$\frac{\left(\frac{-8x-7}{\cancel{x-1}}\right)\frac{\cancel{(x-1)}}{1} - 7\frac{(x-1)}{1}}{\left(\frac{-8x-7}{\cancel{x-1}}\right)\frac{\cancel{(x-1)}}{1} + 8\frac{(x-1)}{1}} = \frac{-8x\cancel{-7}-7x\cancel{+7}}{\cancel{-8x}-7\cancel{+8x}-8}$$

$$\frac{-\cancel{15}x}{-\cancel{15}} = x \checkmark$$

- Right side ~~f(g(x))~~ g(f(x))

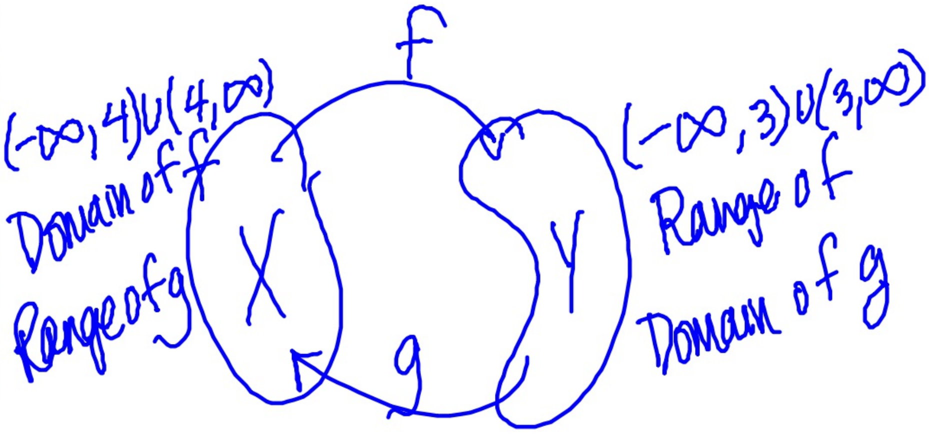
$$g\left(\frac{x-7}{x+8}\right)=\frac{\left(-8\left(\frac{x-7}{x+8}\right)-7\right)\frac{x+8}{1}}{\left(\left(\frac{x-7}{x+8}\right)-1\right)\frac{x+8}{1}}$$

$$\frac{-8\left(\frac{x-7}{\cancel{x+8}}\right)\frac{\cancel{(x+8)}}{1}-7\frac{(x+8)}{1}}{\left(\frac{x-7}{\cancel{x+8}}\right)\frac{\cancel{(x+8)}}{1}-1\frac{(x+8)}{1}}=\frac{-8x+\cancel{56}-7x-\cancel{56}}{\cancel{x}-7-\cancel{x}-8}=\frac{-15x}{-15}=x \checkmark$$

Since $f(g(x))$ = x and $g(f(x))$ = x, we have proven f(x) and g(x) are inverses.

$$f(x)=\frac{3x+2}{x-4}$$

$$g(x)=\frac{4x+2}{x-3}$$



$$\boxed{f(g(x))=x} \qquad g(f(x))=x$$

$$f\left(\frac{4x+2}{x-3}\right)=\frac{\left(3\left(\frac{4x+2}{x-3}\right)+2\right)}{\left(\left(\frac{4x+2}{x-3}\right)-4\right)}\cdot\frac{\frac{x-3}{1}}{\frac{x-3}{1}}$$

$$\frac{3\left(\frac{4x+2}{\cancel{x-3}}\right)\frac{\cancel{(x-3)}}{1} + 2\frac{(x-3)}{1}}{\left(\frac{4x+2}{\cancel{x-3}}\right)\frac{\cancel{(x-3)}}{1} - 4\frac{(x-3)}{1}} = \frac{12x+\cancel{6}+2x-\cancel{6}}{\cancel{4x}+2-\cancel{4x}+12}$$

$$\frac{14x}{14} = \boxed{x}$$

$$f(x)=\frac{3x+2}{x-4}$$

$$g(x)=\frac{4x+2}{x-3}$$

$$g\left(\frac{3x+2}{x-4}\right)$$

$$\left(\frac{4\left(\frac{3x+2}{x-4}\right)+2}{\left(\frac{3x+2}{x-4}\right)-3}\right)\frac{(x-4)}{1}\frac{(x-4)}{1}$$

$g(f(x))$

$$\frac{4\left(\frac{3x+2}{\cancel{x-4}}\right)\frac{\cancel{(x-4)}}{1}+2\frac{(x-4)}{1}}{\left(\frac{3x+2}{\cancel{x-4}}\right)\frac{\cancel{(x-4)}}{1}-3\frac{(x-4)}{1}} = \frac{12x+\cancel{8}+2x-\cancel{8}}{\cancel{3x}+2-\cancel{3x}+12}$$

$$\frac{14x}{14}=\boxed{x}$$

Since $f(g(x))$ = x and $g(f(x))$ = x, we have proven f(x) and g(x) are inverses.