Today's Objectives

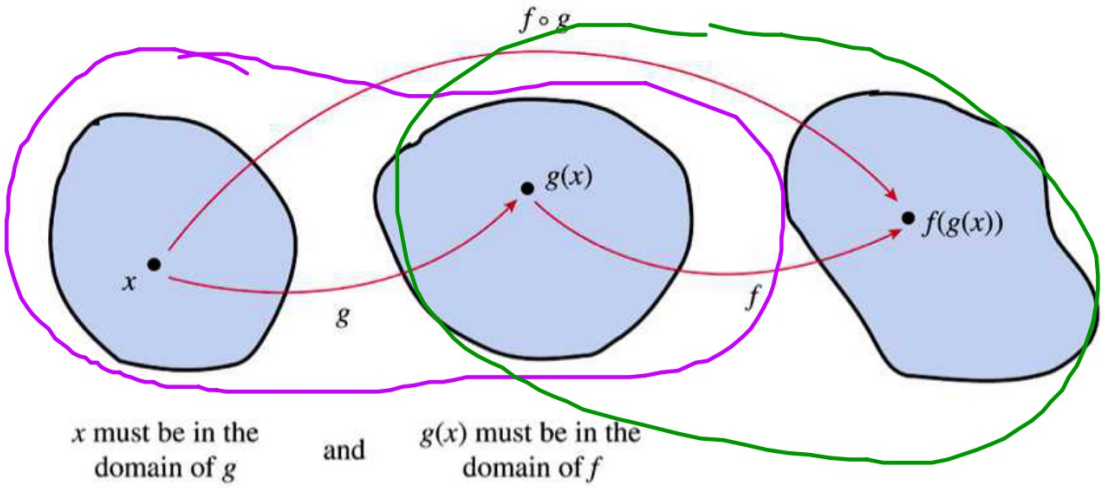
- **Produce new functions by composing existing functions and evaluate for given values** after listening to a step-by-by explanation with key words.
- Success Criteria
  - Define composition and notation
  - Assess functions for compatible domains and ranges
- Vocabulary: composition, compatible

Composition of Functions

Let f and g be two functions such that the domain of f intersects the range of g. The composition f of g, denoted $f \circ g$, is defined by the rule $(f \circ g)(x) = f(g(x))$.

The domain of $f \circ g$ consists of all x-values in the domain of g that map to $g(x)$-values in the domain of f.

Composition of Functions



x must be in the domain of g and $g(x)$ must be in the domain of f

-3

Domain: What you can put into your function

Dry, dirty laundry

Range: What comes out of your function

This is also the domain for your second function!!

Clean, wet laundry

The washing machine is like your 1st function

The dryer is like your 2nd function

Dry, clean laundry

Range: What comes out of your function

This is range for your composite function!!

Example Composing Functions

Let $f(x) = 2^x$ and $g(x) = \sqrt{x+1}$. Find

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

a) $f(g(x)) = f(\sqrt{x+1}) = 2^{\sqrt{x+1}}$

$f(x) = 2^{(x)}$

b) $g(f(x)) = g(2^x) = \sqrt{2^x+1}$

$g(x) = \sqrt{(x)+1}$

Example Composing Functions

Let $f(x) = -x^2 + 4$ and $g(x) = \sqrt{x}$. Find

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

a) $f(g(x)) = f(\sqrt{x})$

$f(x) = -x^2 + 4$ $\quad = -(\sqrt{x})^2 + 4$

$-x + 4$

b) $g(f(x)) = g(-x^2 + 4)$

$g(x) = \sqrt{x}$ $\quad = \sqrt{-x^2 + 4}$

AM: Find compositions of 2 functions

1. If $f(x) = x^4$ and $g(x) = 1-2x^2$, find $g(f(x))$.

[A] $1-2x^8$ [B] $\frac{x^4}{1-2x^2}$ [C] $(1-2x^2)^4$ [D] x^4-2x^6

$g(x^4) = 1-2(x^4)^2$

$= 1-2x^8$

$g(x) = 1-2x^2$

AM: Find compositions of 2 functions

2. Given $f(x) = \frac{x+5}{x}$ and $g(x) = x^2 + 4$, find $(g \circ f)(6)$.

[A] $\frac{520}{121}$ [B] $\frac{9}{8}$ [C] $\frac{265}{36}$ [D] $\frac{35}{6}$

$g(f(6)) = g\left(\frac{11}{6}\right) = \left(\frac{11}{6}\right)^2 + 4$

① $f(6) = \frac{6+5}{6} = \frac{11}{6}$

$= \frac{121}{36} + 4 \cdot \frac{36}{36}$

$\frac{121}{36} + \frac{144}{36} = \frac{265}{36}$

AM: Find Composition of 2 functions

3. Given $f(x) = -2x^2$, $g(x) = -3x + 7$, and $h(x) = \sqrt{x}$, find $\left[(f+g) \circ h\right](x)$.

[A] $-2x - 3\sqrt{x} + 7$ [B] $-2x^2 - 3\sqrt{x} + 7$ [C] $6x + \sqrt{x} + 7$ [D] $-2\sqrt{x} - 3x + 7$

$[(f+g) \circ h](x)$

① $f(x) + g(x)$

$-2x^2 + -3x + 7$

$B(x) = -2x^2 - 3x + 7$

$(B \circ h)(x)$

$B(\sqrt{x})$

$-2(\sqrt{x})^2 - 3(\sqrt{x}) + 7$

$-2x - 3\sqrt{x} + 7$