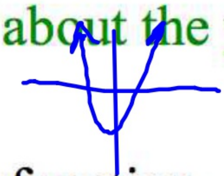
Page 98 Example 9 (a)
Checking Functions for Symmetry about the y-axis

$f(-x) = (-x)^2 - 3 = x^2 - 3$



Conjecture: $f(x) = x^2 - 3$ is an even function, because the graph of f is a parabola that is symmetric about the y-axis.

Steps for an Algebraic Proof: Even functions satisfy the relationship $\boxed{f(x) = f(-x)}$, therefore we must determine a rule for $f(-x)$. Compare the rules for $f(-x)$ and $f(x)$ to determine if the relationship holds.

In order to prove $f(x)$ is even, I must show that $f(x) = f(-x)$.

$f(x) = x^2 - 3$ $\qquad$ $f(-x) = (-x)^2 - 3$

$= x^2 - 3$

Since I have shown $f(x) = f(-x) = x^2 - 3$, I can say that the function is even. $\square$

Q.E.D.

p. 98 Example 9 (c)
Checking Functions for Origin Symmetry

Find the rules for *f(-x)* and *–f(x)*.

$$f(x) = \frac{x^3}{4 - x^2}$$

$$\underbrace{-x \cdot -x}_{} \cdot -x$$

$$f(-x) = \frac{(-x)^3}{4-(-x)^2} = \frac{-x^3}{4-x^2}$$

$$-f(x) = \frac{-1}{1}\left(\frac{x^3}{4-x^2}\right) = \frac{-x^3}{4-x^2}$$

Practice

a) Since the function f(x) has symmetry with respect to the origin, we can make the conjecture that f(x) is an odd function.

b) In order to prove that f(x) is an odd function, we must show that f(-x) = -f(x).

Since f(-x) = -f(x), we have shown that f(x) is an odd function.

$$f(x) = \frac{x^3}{4 - x^2}$$

Conjecture: This an _______ function, because the graph has symmetry with respect to the origin. Origin symmetry mean that the reflection of the graph of *f* over the ________ or *f(-x)* is equivalent to the reflection of the graph of *f* over the ________ or *–f(x)*.

Proof: ______ functions satisfy the algebraic relationship________, therefore we must determine rules for______ and ______. By comparing the rules for _______and______ we have determine *f(x)* is _____ because the relationship _____________holds.

Example Checking Functions for Symmetry

Tell whether the following function is odd, even, or neither.

$f(x) = x^2 + 3$ $f(-x) = (-x)^2 + 3 = x^2 + 3$

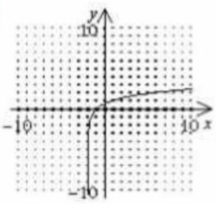
Conjecture: This is an even function, because the graph of f is a parabola that is symmetric about the y-axis.

Steps for an Algebraic Proof: Even functions satisfy the algebraic relationship $f(x) = f(-x)$, therefore we must determine a rule for $f(-x)$. Compare the rules for $f(x)$ and $f(-x)$ to determine if the relationship holds.

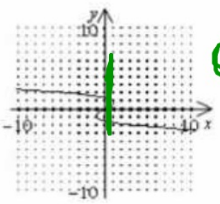
AM: Classify Functions as even odd, or neither

1. Which is the graph of *neither* an even nor an odd function?

[A]

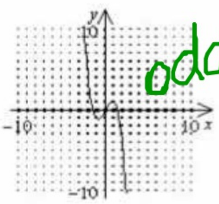


[B]



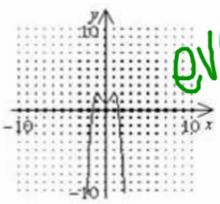
odd

[C]



odd

[D]



even

Conjecture: The function______ is even, because the graph of f is symmetric about the______________.

AM: Classify Functions as even odd, or neither

2. Which of the following functions is even?

6 [A] $f(x) = 6x^6 + 2x^2 - 5$ [B] $g(x) = |6x+2| - 6$

[C] $F(x) = \dfrac{6x^3}{6x^2+5}$ [D] $h(x) = 5x^7 + 2x^3$

$f(-x) = 6(-x)^6 + 2(-x)^2 - 5$

$= 6x^6 + 2x^2 - 5$

Conjecture: The function______ is even, because the graph of f is symmetric about the______________.

Proof: Even functions satisfy the algebraic relationship_________, therefore we must algebraically determine a rule for______. By comparing the rules for ________and______ we determined they are equivalent, which proves our conjecture that ______ is an even function.

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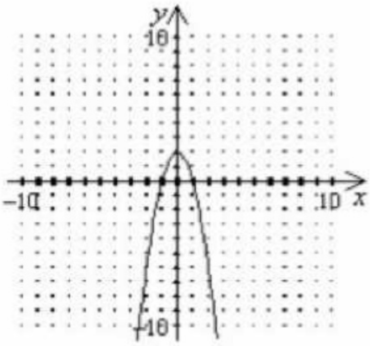
AM: Classify Functions as even odd, or neither

3. Use the graph to determine if the function is odd, even, or neither.



Conjecture: The function______ is even, because the graph of f is symmetric about the_____________.

Proof: Even functions satisfy the algebraic relationship________, therefore we must algebraically determine a rule for______. By comparing the rules for _______and______ we determined they are equivalent, which proves our conjecture that ______ is an even function.

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Practice

a) Since the function _____ has symmetry with respect to the _______, we can make the conjecture that _____ is a ______ function.

b) In order to prove that _____ is a ______ function, we must show that $f(-x)$ = $-f(x)$.

$f(-x) = \frac{1}{5}(-x)^5 - (-x)^3 = -\frac{1}{5}x^5 + x^3$

$-f(x) = -1\left(\frac{1}{5}x^5 - x^3\right) = -\frac{1}{5}x^5 + x^3$ ✓

Since $f(-x)$ = $-f(x)$, we have shown that ______ is a ______ function.