Today's Objectives

- Predict and describe in <u>writing</u> **the resulting graph by analyzing rigid and non-rigid graph transformations** using key words in a small group.
- Success Criteria
  - Define and identify horizontal shifts
  - Define and identify vertical shifts
  - Define and identify reflections about the axis
  - Define and identify stretches and shrinks
- Vocabulary: horizontal shift, vertical shift, reflection, stretch, shrink

Translations = Rigid Transformations
= Shifts

Let c be a positive real number. Then the following transformations result in translations of the graph of $y=f(x)$.

Horizontal Translations

$y=f(x-c)$ a translation to the right by c units

$y=f(x+c)$ a translation to the left by c units

Vertical Translations

$y=f(x)+c$ a translation up by c units

$y=f(x)-c$ a translation down by c units

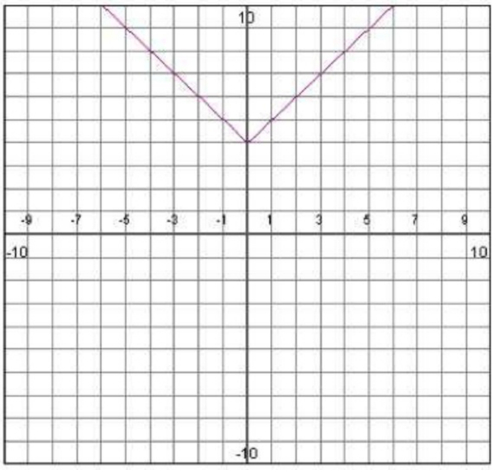
Example Vertical Translations

Describe how the graph of $f(x) = |x|$ can be transformed to the graph of $y = |x| + 4$.

Example Vertical Translations

Describe how the graph of $f(x) = |x|$ can be transformed to the graph of $y = |x| + 4$.

The equation is in the form $y = f(x) + 4$, a translation up by 4 units.

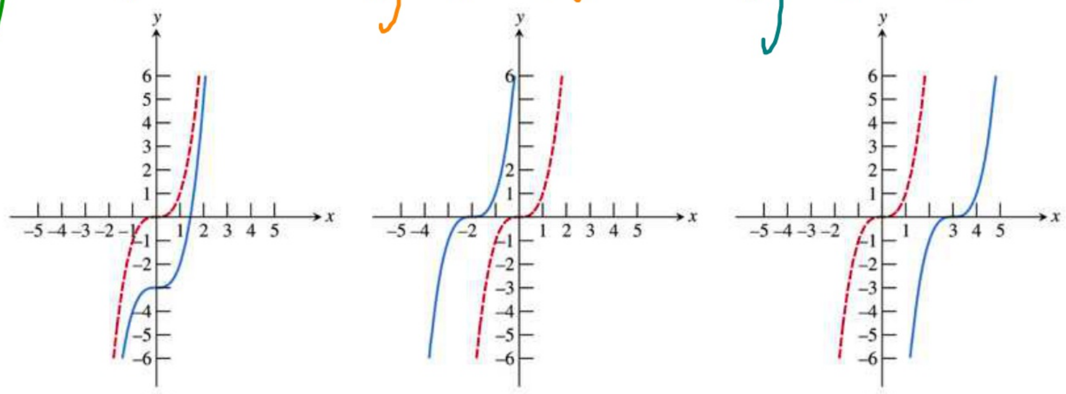


Example Finding Equations for Translations

$y=x^3$

Each view shows the graph of $y_1 = x^3$ and a vertical or horizontal translation y_2. Write an equation for y_2.

$y=(x)^3-3$ $y=(x+2)^3$ $y=(x-3)^3$



(a) (b) (c)

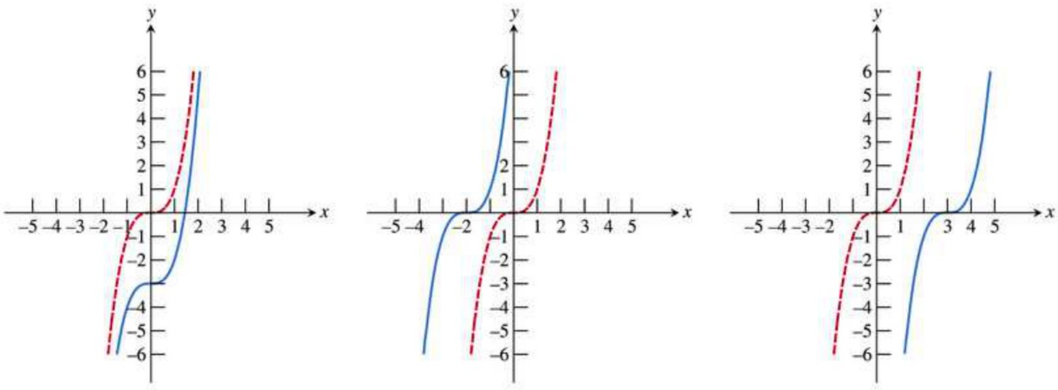
Example Finding Equations for Translations

Each view shows the graph of $y_1 = x^3$ and a vertical or horizontal translation y_2. Write an equation for y_2.

(a) $y_2 = x^3 - 3$

(b) $y_2 = (x+2)^3$

(c) $y_2 = (x-3)^3$



(a) (b) (c)

Reflections

The following transformations result in reflections of the graph of $y = f(x)$:

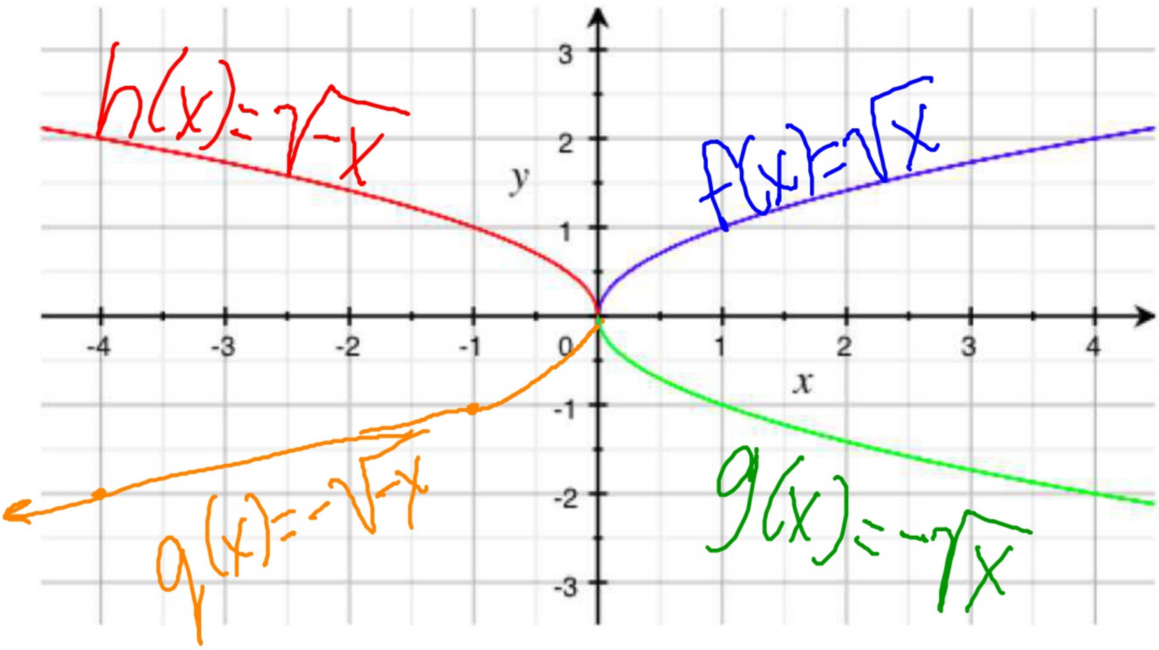
Across the x-axis

$y = -f(x)$

Across the y-axis

$y = f(-x)$

Example Finding Equations for Reflections



Graphing Absolute Value Compositions

Given the graph of y = f(x),

the graph $g(x) = |f(x)|$ can be obtained by reflecting the portion of the graph below the x-axis across the x-axis, leaving the portion above the x-axis unchanged;

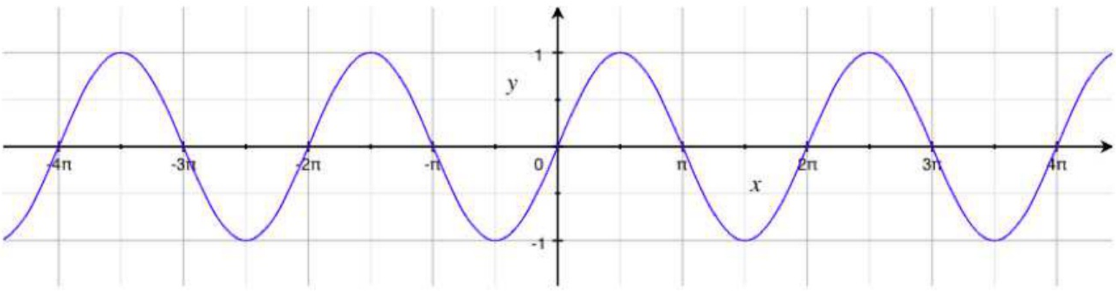
the graph of $h(x) = f(|x|)$ can be obtained by replacing the portion of the graph to the left of the y-axis by a reflection of the portion to the right of the y-axis across the y-axis, leaving the portion to the right of the y-axis unchanged. (The result will show even symmetry.)

Absolute Value Compositions

$$f(x) = \sin(x)$$

$$h(x) = \quad f(\qquad)$$

$$g(x) = \quad f(\qquad)$$



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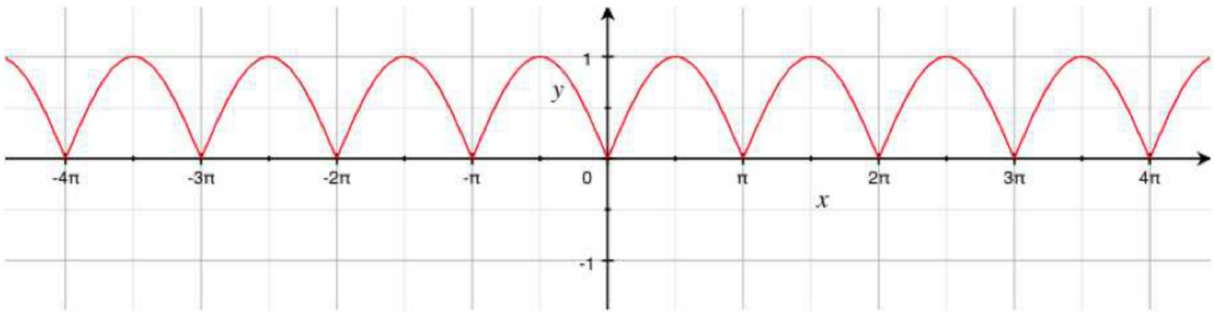

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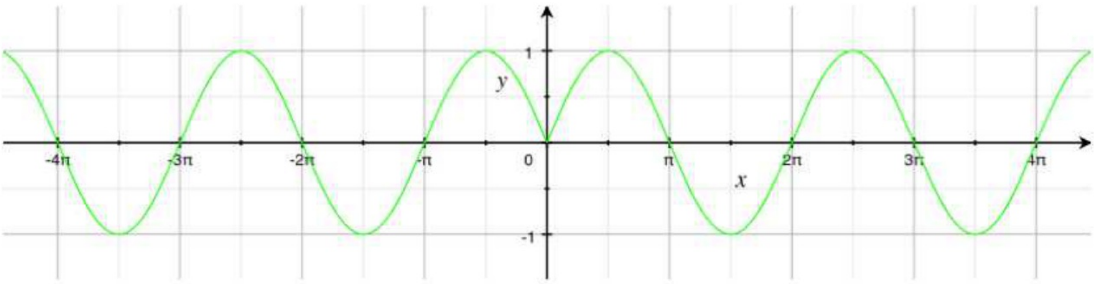


Stretches and Shrinks

* non-rigid transformation

Let c be a positive real number. Then the following transformations result in stretches or shrinks of that graph $y = f(x)$:

Horizontal Stretches or Shrinks

$$y = f\left(\frac{x}{c}\right) \quad \begin{cases} \text{a stretch by a factor of } c \text{ if } c > 1 \\ \text{a shrink by a factor of } c \text{ if } c < 1 \end{cases}$$

Vertical Stretches or Shrinks

$$y = c \cdot f(x) \quad \begin{cases} \text{a stretch by a factor of } c \text{ if } c > 1 \\ \text{a shrink by a factor of } c \text{ if } c < 1 \rightarrow 0 < c < 1 \end{cases}$$

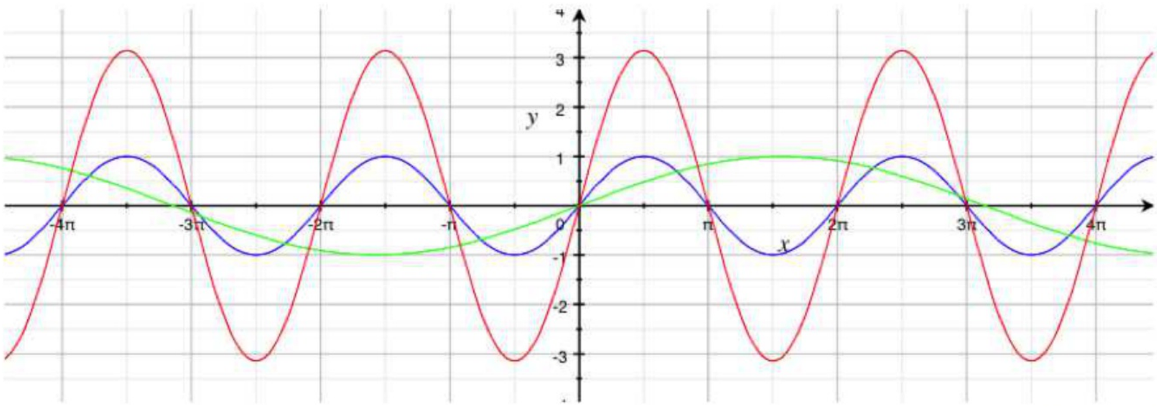
Stretches and Shrinks

$c = \pi$

$f(x) = \sin(x)$

$h(x) = f\left(\frac{x}{3}\right) = \sin\left(\frac{x}{3}\right)$

$g(x) = 3\, f(x) = 3\sin(x)$



What order do I apply transformations?

- Remember HSRV
  - H - apply horizontal shifts
  - S - apply stretches or shrinks
  - R - apply reflections across the x or y axis
  - V - apply the vertical shifts

  - Basically, you work from the inside to the outside of the function.

Example Finding Equations for Stretches and Shrinks

Let C_1 be the curve defined by $y_1 = x^3 + 3$. Find equations for the following non-rigid transformations of C_1:

(a) C_2 is a vertical stretch of C_1 by a factor of 4.

(b) C_3 is a horizontal shrink of C_1 by a factor of 1/3.

Example Finding Equations for Stretches and Shrinks

Let C_1 be the curve defined by $y_1 = x^3 + 3$. Find equations for the following non-rigid transformations of C_1:

(a) C_2 is a vertical stretch of C_1 by a factor of 4.

(b) C_3 is a horizontal shrink of C_1 by a factor of 1/3.

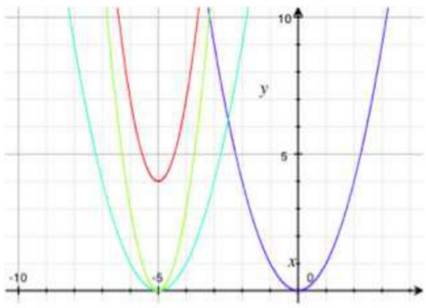
$$\text{(a) } y_2 = 4 \times f(x) = 4(x^3 + 3) = 4x^3 + 12$$

$$\text{(b) } y_3 = f\left(\frac{x}{1/3}\right) = f(3x) = (3x)^3 + 3 = 27x^3 + 3$$

Example Combining Transformations in Order

The graph of $y = x^2$ undergoes the following transformations, in order. Find the equation of the graph that results.

- a horizontal shift 5 units to the left
- a vertical stretch by a factor of 3
- a vertical translation 4 units up



Define $f(x) = y$.

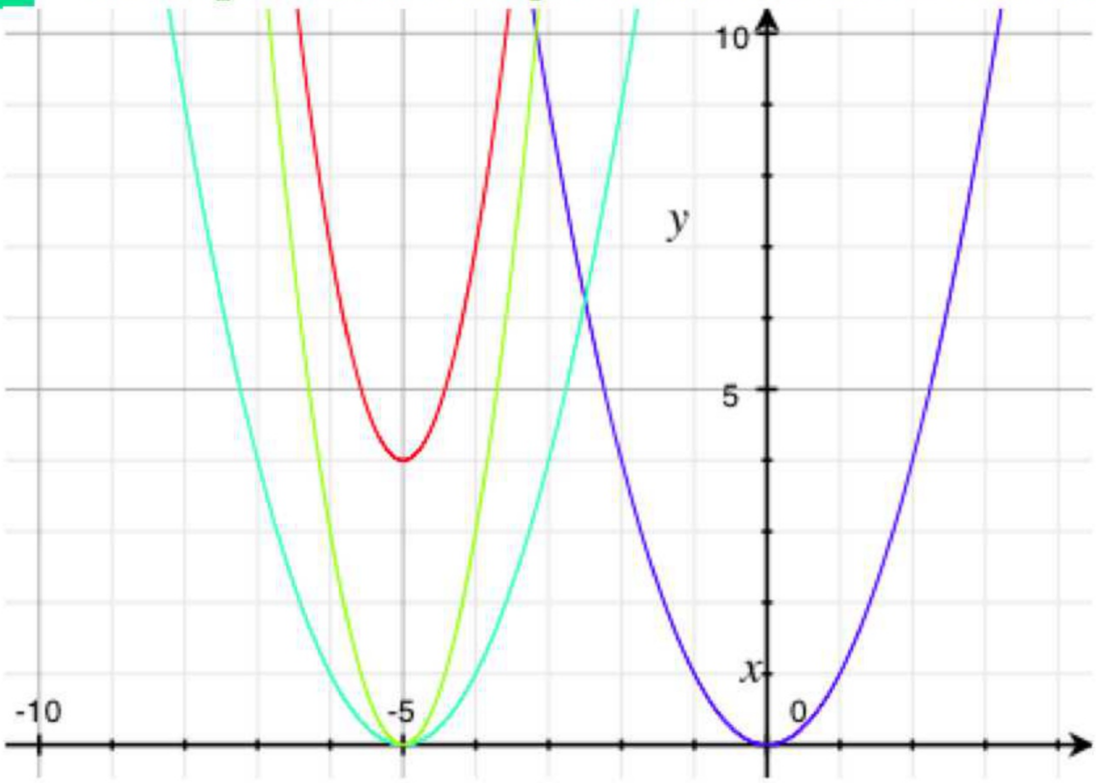
$C_1 : f(x) = x^2.$

$C_2 : g(x) = \quad f(x+5) = (x+5)^2$

$C_3 : h(x) = 3g(x+5) = 3(x+5)^2$

$C_4 : j(x) = 3h(x+5)+4 = 3(x+5)^2$

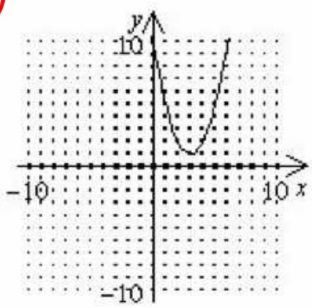
Example Combining Transformations in Order



AM:Use Rigid and Non-Rigid Transformations to Graph Functions

1. Graph: $y = (x - 3)^2 + 1$ R Up

[A]



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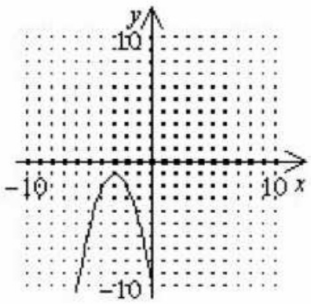

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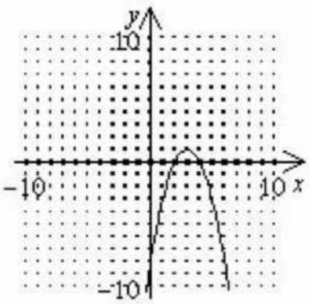

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