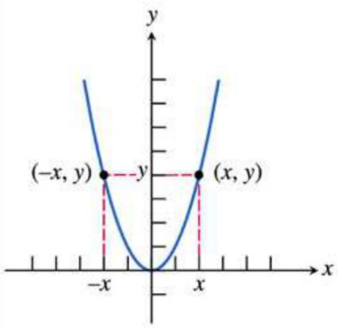
Symmetry with respect to the y-axis

Example: $f(x) = x^2$

Graphically



Numerically

x	$f(x)$
-3	9
-2	4
-1	1
1	1
2	4
3	9

Algebraically

For all x in the domain of f,

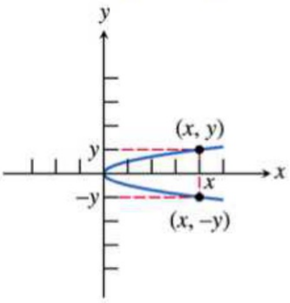
$$f(-x) = f(x)$$

Functions with this property (for example, x^n, n even) are **even** functions.

Symmetry with respect to the x-axis

Example: $x = y^2$

Graphically



Numerically

x	y
9	−3
4	−2
1	−1
1	1
4	2
9	3

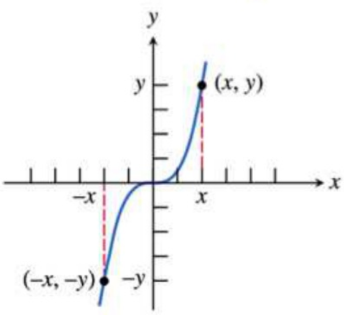
Algebraically

Graphs with this kind of symmetry are not functions (except the zero function), but we can say that $(x, -y)$ is on the graph whenever (x, y) is on the graph.

Symmetry with respect to the origin

Example: $f(x) = x^3$

Graphically



Numerically

x	y
−3	−27
−2	−8
−1	−1
1	1
2	8
3	27

Algebraically

For all x in the domain of f,

$$f(-x) = -f(x).$$

Functions with this property (for example, x^n, n odd) are **odd** functions.

Today's Objectives

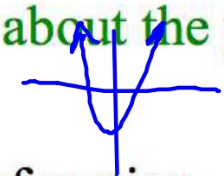
- Justify **a conjecture of the classification of different functions as even, odd, or neither** by writing algebraic proofs with a partner.
- Success Criteria
  - Identify different kinds of graph symmetry
  - Use algebraic proofs to defend classification
- Vocabulary: even, odd, neither, symmetry

Page 98 Example 9 (a)
Checking Functions for Symmetry about the y-axis

- Reproduce Figure 1.29 by graphing $f(x) = x^2 - 3$ on your calculator.
- Set the window ranges as given in the textbook.
- Make a conjecture about the symmetry of f.

Page 98 Example 9 (a)
Checking Functions for Symmetry about the y-axis

$f(-x) = (-x)^2 - 3 = x^2 - 3$



Conjecture: $f(x) = x^2 - 3$ is an even function, because the graph of f is a parabola that is symmetric about the y-axis.

Steps for an Algebraic Proof: Even functions satisfy the relationship $\boxed{f(x) = f(-x)}$, therefore we must determine a rule for $f(-x)$. Compare the rules for $f(-x)$ and $f(x)$ to determine if the relationship holds.

In order to prove $f(x)$ is even, I must show that $f(x) = f(-x)$.

$f(x) = x^2 - 3$ $\qquad$ $f(-x) = (-x)^2 - 3$

$= x^2 - 3$

Since I have shown $f(x) = f(-x) = x^2 - 3$, I can say that the function is even. $\square$

Q.E.D.

p. 98 Example 9 (c)
Checking Functions for Origin Symmetry

Find the rules for *f(-x)* and *–f(x)*.

$$f(x) = \frac{x^3}{(4 - x^2)}$$

$-x \cdot +x \cdot +x$

$$f(-x) = \frac{(-x)^3}{4-(-x)^2} = \frac{-x^3}{4-x^2}$$

$$-1 \cdot f(x) = -1\left(\frac{x^3}{4-x^2}\right) = \frac{-x^3}{4-x^2}$$

Since $f(-x) = -f(x)$ we can say the function is odd.

$$f(x) = \frac{x^3}{4 - x^2}$$

Conjecture: This an _______ function, because the graph has symmetry with respect to the origin. Origin symmetry mean that the reflection of the graph of *f* over the ________ or *f(-x)* is equivalent to the reflection of the graph of *f* over the ________ or *–f(x)*.

Proof: ______ functions satisfy the algebraic relationship________, therefore we must determine rules for______ and ______. By comparing the rules for _______and______ we have determine *f(x)* is _____ because the relationship _____________holds.

Example Checking Functions for Symmetry

Tell whether the following function is odd, even, or neither.

$f(x) = x^2 + 3$ $\quad f(-x) = (-x)^2 + 3 = x^2 + 3$

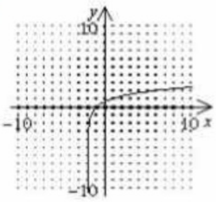
Conjecture: This is an even function, because the graph of f is a parabola that is symmetric about the y-axis.

Steps for an Algebraic Proof: Even functions satisfy the algebraic relationship $f(x) = f(-x)$, therefore we must determine a rule for $f(-x)$. Compare the rules for $f(x)$ and $f(-x)$ to determine if the relationship holds.

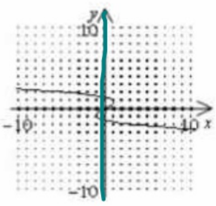
AM: Classify Functions as even odd, or neither

1. Which is the graph of *neither* an even nor an odd function?

[A]

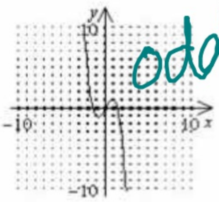


[B]



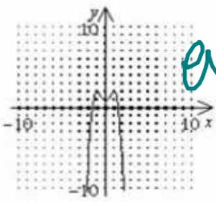
odd

[C]



odd

[D]



even

Conjecture: The function______ is even, because the graph of f is symmetric about the_____________.

AM: Classify Functions as even odd, or neither

2. Which of the following functions is even?

[A] $f(x) = 6x^6 + 2x^2 - 5$ [B] $g(x) = |6x+2| - 6$

~~[C]~~ $F(x) = \dfrac{6x^3}{6x^2+5}$ ~~[D]~~ $h(x) = 5x^7 + 2x^3$

$$f(-x) = 6(-x)^6 + 2(-x)^2 - 5 = 6x^6 + 2x^2 - 5$$

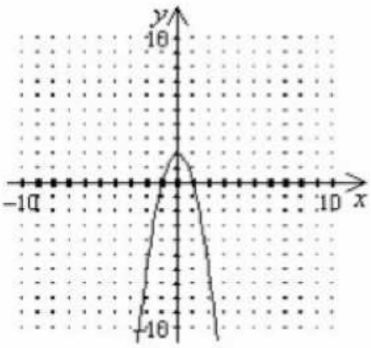
Conjecture: The function $f(x)$ is even, because the graph of f is symmetric about the y-axis.

Proof: Even functions satisfy the algebraic relationship $f(x) = f(-x)$, therefore we must algebraically determine a rule for $f(-x)$. By comparing the rules for $f(x)$ and $f\cdot(x)$ we determined they are equivalent, which proves our conjecture that $f(x)$ is an even function.

8

AM: Classify Functions as even odd, or neither

3. Use the graph to determine if the function is odd, even, or neither.



Conjecture: The function______ is even, because the graph of f is symmetric about the_____________.

Proof: Even functions satisfy the algebraic relationship________, therefore we must algebraically determine a rule for______. By comparing the rules for _______and______ we determined they are equivalent, which proves our conjecture that ______ is an even function.

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$f(x) = \frac{1}{5}x^5 - x^3$

Practice

a) Since the function __f(x)__ has symmetry with respect to the __origin__, we can make the conjecture that __f(x)__ is a__n odd__ function.

b) In order to prove that __f(x)__ is a__n odd__ function, we must show that __f(-x)__ = __-f(x)__.

$f(-x) = \frac{1}{5}(-x)^5 - (-x)^3 = -\frac{1}{5}x^5 + x^3$

$-f(x) = -1\left(\frac{1}{5}x^5 - x^3\right) = -\frac{1}{5}x^5 + x^3$

Since __f(-x)__ = __-f(x)__, we have shown that __f(x)__ is a__n odd__ function. $\square$