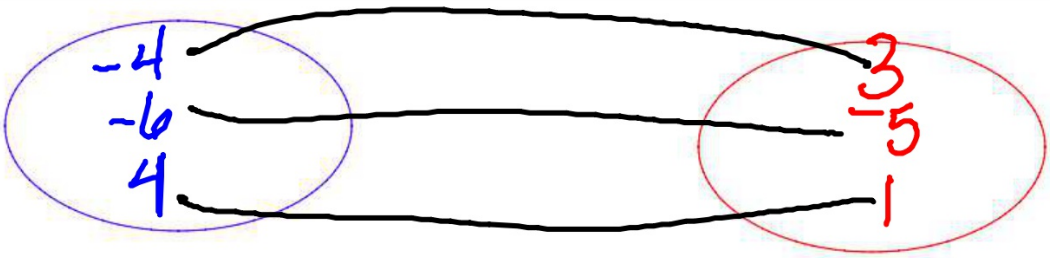
AM: Domain and range, relations and functions

1. Find the domain of the relation $\{(-4, 3), (-6, -5), (4, 1)\}$.

[A] $\{-4, -6, 4\}$ [B] $\{3, -5, 4\}$ [C] $\{3, -5, 1\}$ [D] $\{-4, -6, 1\}$



LO: The domain is the set of numbers
{ -4, -6, 4 }, because those are the inputs,
or x values of the relation.

AM: Domain and range, relations and functions

Find the domain and range:

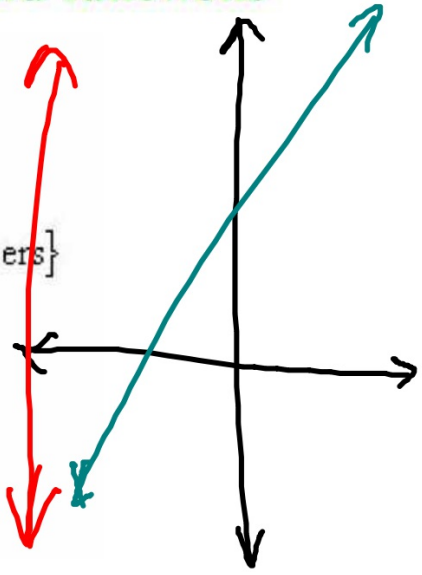
2. $\{(x, y) | y = 7 + x\}$

[A] $D = \{\text{all real numbers}\}$; $R = \{\text{all real numbers}\}$

[B] $D = \{-7 \le x \le 7\}$; $R = \{\text{all real numbers}\}$

[C] $D = \{\text{all real numbers}\}$; $R = \{x | x \ge -7\}$

[D] $D = \{x | x \ge -7\}$; $R = \{\text{all real numbers}\}$



LO: The domain is the set of numbers {(-∞, ∞)}, because those are the inputs or x values of the relation.

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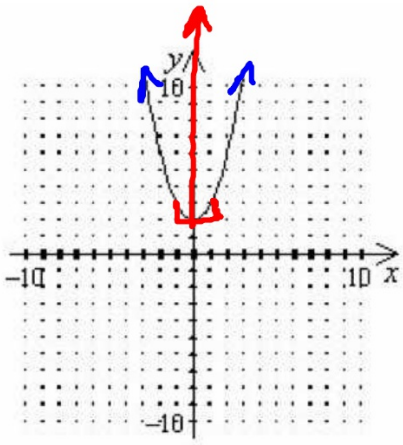
3.



LO: The domain is the set of numbers { $(-\infty, \infty)$ }, because those are the inputs, or x values of the relation. The range is the set of numbers { $[2, \infty)$ }, because those are the output, or y values of the relation.

[A] $D = \{y \mid y \leq 2\}$
$R = \{y \mid y \text{ is a real number}\}$

[B] $D = \{x \mid x \text{ is a real number}\}$
$R = \{y \mid y \text{ is a real number}\}$

[C] $D = \{x \mid x \text{ is a real number}\}$
$R = \{y \mid y \geq 2\}$

[D] $D = \{x \mid x > 2\}$
$R = \{y \mid y > 2\}$

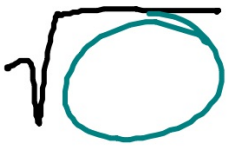
Agreement

Unless we are dealing with a model that necessitates a restricted domain, we will assume that the domain of a function defined by an algebraic expression is the same as the domain of the algebraic expression, the implied domain.

For models, we will use a domain that fits the situation, the relevant domain.

Process for Finding a Domain

- Look at the functions for things that cause mathematical problems.
  - Do I have any square roots? (Can't be negative)
  - Do I have a denominator? (Can't be zero)
- Set up equation/inequality
  - For square roots, the radicand ≥ 0
  - For fractions, the denominator = 0
- Write answer using interval notation
  - *Remember you can put intervals together by writing a U between them (U means 'union')



Example Finding the Domain of a Function

Find the domain of the function.

$f(x) = \sqrt{x+2}$

$x+2 \geq 0$

$-2 \quad -2$

$x \geq -2$

$x \geq -2$

$[-2, \infty)$

$y \geq 0$

$[0, \infty)$

Example Finding the Domain of a Function

Find the domain of the function.

$f(x) = \sqrt{x+2}$

Solve algebraically:

The expression under a radical may not be negative.

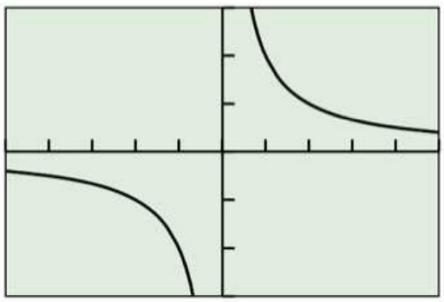
$x + 2 \geq 0$

$x \geq -2$

The domain of f is the interval $[-2, \infty)$.

Example Finding the Range of a Function

Find the range of the function $f(x) = \frac{2}{x}$.



[–5, 5] by [–3, 3]

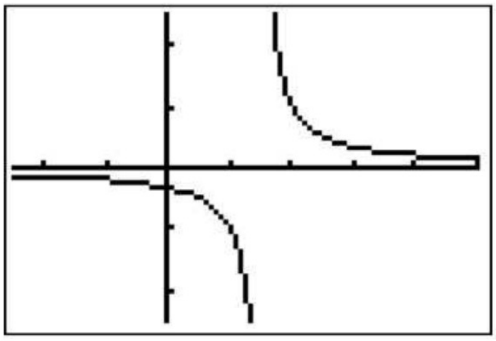
Solve Graphically:

The graph of $y = \frac{2}{x}$ shows that the range is all real numbers except 0.

The range in interval notation is $(-\infty, 0) \cup (0, \infty)$.

Example Finding the Domain and Range of a Function

Find the domain and range of the function $f(x) = \dfrac{1}{2x-3}$.



Solve Graphically:

The graph of $y = \dfrac{2}{2x-3}$ shows that the domain is all real numbers except $\dfrac{3}{2}$.

The domain in interval notation is $\left(-\infty, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$.

Example Finding the Domain and Range of a Function

Find the domain and range of the function $f(x) = \dfrac{1}{2x-3}$.

$2x-3 \neq 0$

$+3 \quad +3$

$2x \neq 3$

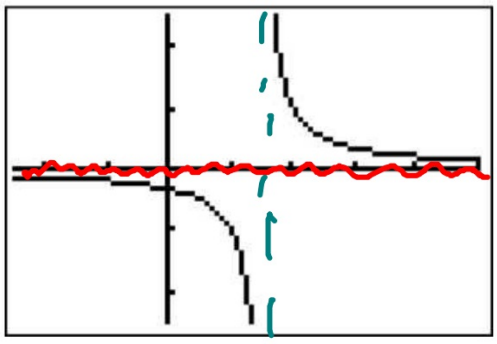
$\dfrac{2x}{2} \neq \dfrac{3}{2}$

$x \neq \dfrac{3}{2}$

D: $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

How can we find the domain analytically, that means without looking at a graph or table?



Example Finding the Domain and Range of a Function

Find the domain and range of the function $f(x) = \sqrt{2x+5} - 7$

$2x+5 \geq 0$

$-5 \quad -5$

$\frac{2x}{2} \geq \frac{-5}{2}$

$x \geq -\frac{5}{2}$

$D: \left[-\frac{5}{2}, \infty\right)$

$R: [-7, \infty)$

AM: Domain & range, functions

1. Find the domain and range: $y = \sqrt{x-6} + 8$

[A] domain: $\{x \mid x \geq 6\}$, range: $\{y \mid y \geq 8\}$

[B] domain: $\{x \mid x \geq 0\}$, range: $\{y \mid y \geq 8\}$

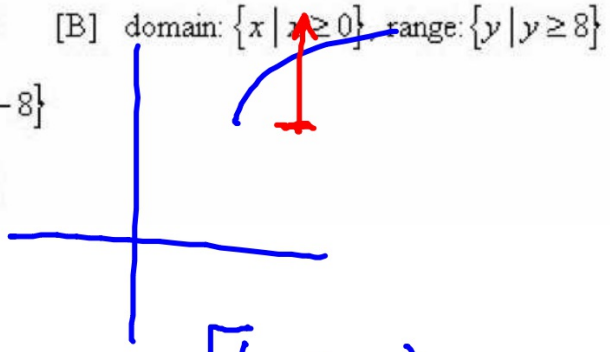
[C] domain: $\{x \mid x \geq -6\}$, range: $\{y \mid y \geq -8\}$

[D] domain: $\{x \mid x \geq 0\}$, range: $\{y \mid y \geq 0\}$

$x - 6 \geq 0$

$+6 \quad +6$

$x \geq 6$



LO: The domain is the set of numbers { $[6, \infty)$ }, because those are the inputs, or x values of the relation.

The range is the set of numbers { $[8, \infty)$ }, because those are the output, or y values of the relation.

AM: Domain & range, functions

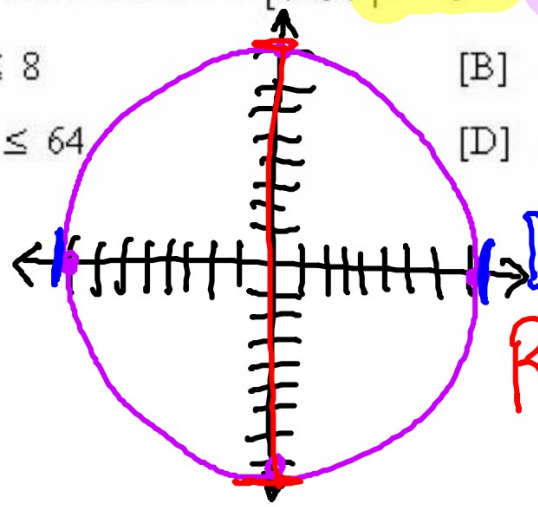
2. Find the range of the relation $A = \{(x,y) \mid x^2 + y^2 = 64\}$.

[A] $-8 \le y \le 8$

[B] $-64 \le y \le 64$

[C] $-64 \le x \le 64$

[D] $-8 \le x \le 8$



D: $-8 \le x \le 8$

R: $-8 \le y \le 8$

LO: The range is the set of numbers {$-8 \le y \le 8$}, because those are the _____, or ___ values of the relation.

AM: Domain & range, functions

3. Find the domain and range of the relation $\{(x, y) \mid 2x < 12\}$.

[A] domain = $\mathbf{R}$; range = $\{y \mid y < 12\}$

[B] domain = $\{x \mid x < 6\}$; range = $\mathbf{R}$

[C] domain = $\{x \mid x < 12\}$; range = $\{y \mid y < 6\}$

[D] domain = $\mathbf{R}$; range = $\{y \mid y < 6\}$

$\frac{2x}{2} < \frac{12}{2}$

$x < 6$

LO: The domain is the set of numbers {_____}, because those are the _____, or x values of the relation.

The range is the set of numbers {_____}, because those are the _____, or y values of the relation.

AM: Domain & range, functions

4. Determine the domain: $h(x) = \dfrac{2x}{x(x^2-16)}$

[A] $\{x \mid x \neq \pm 4, x \neq 0\}$ [B] $\{x \mid x \neq \pm 4\}$

[C] $\{x \mid x \neq \pm 16, x \neq 0\}$ [D] $\{x \mid x \neq 4\}$

$x(x^2-16) \neq 0$

$x \neq 0$ $x^2-16 \neq 0$

$x^2-16 \neq 0$

$+16 \quad +16$

$\sqrt{x^2} \neq \sqrt{16}$

$x \neq \pm 4$

LO: The domain is the set of numbers {_____}, because those are the _____, or x values of the relation.