

# 3.6

## Mathematics of Finance

# Quick Review

1. Find 3.4% of 70.
2. What is one-third of 6.25%?
3. 30 is what percent of 150?
4. 28 is 35% of what number?
5. How much does Allyson have at the end of 1 year if she invests \$400 at 3% simple interest?

# Quick Review Solutions

1. Find 3.4% of 70.  $2.38$
2. What is one-third of 6.25%?  $0.0208\bar{3}$
3. 30 is what percent of 150?  $20\%$
4. 28 is 35% of what number?  $80$
5. How much does Allyson have at the end of 1 year if she invests \$400 at 3% simple interest?  $\$412$

# What you'll learn about

- Interest Compounded Annually
- Interest Compounded  $k$  Times per Year
- Interest Compounded Continuously
- Annual Percentage Yield
- Annuities – Future Value
- Loans and Mortgages – Present Value

... and why

The mathematics of finance is the science of letting your money work for you – valuable information indeed!

## Interest Compounded Annually

If a **principal**  $P$  is invested at a fixed **annual** interest rate  $r$ , calculated at the end of each year, then the value of the investment after  $n$  years is

$A = P(1 + r)^n$ , where  $r$  is expressed as a decimal.

## Interest Compounded $k$ Times per Year

Suppose a principal  $P$  is invested at an annual rate  $r$  compounded  $k$  times a year for  $t$  years. Then  $\frac{r}{k}$  is the interest rate per compounding period, and  $kt$  is the number of compounding periods. The amount  $A$  in the account after  $t$  years is  $A = P \left( 1 + \frac{r}{k} \right)^{kt}$ .

## Example Compounding Monthly

Suppose Paul invests \$400 at 8% annual interest compounded monthly. Find the value of the investment after 5 years.

Turn and Talk:

Restate the question to construct a sentence, which conveys the real world meaning of your solution.

## Example Compounding Monthly

Suppose Paul invests \$400 at 8% annual interest compounded monthly. Find the value of the investment after 5 years.

Let  $P = 400$ ,  $r = 0.08$ ,  $k = 12$ , and  $t = 5$ ,

$$\begin{aligned} A &= P \left( 1 + \frac{r}{k} \right)^{kt} \\ &= 400 \left( 1 + \frac{0.08}{12} \right)^{12(5)} \\ &= 595.9382\dots \end{aligned}$$

So the value of Paul's investment after 5 years is \$595.94.



# Compound Interest – Value of an Investment

Suppose a principal  $P$  is invested at a fixed annual interest rate  $r$ . The value of the investment after  $t$  years is

$$\bullet A = P \left( 1 + \frac{r}{k} \right)^{kt} \quad \text{when interest compounds } k \text{ times per year,}$$

$$\bullet A = Pe^{rt} \quad \text{when interest compounds continuously.}$$

## Example Compounding Continuously

Suppose Paul invests \$400 at 8% annual interest compounded continuously. Find the value of his investment after 5 years.

Turn and Talk:

1. How is Paul's investment different from his last investment?
2. Make a conjecture about, which type of compounding is gives a better return and be prepared provide your rational.

## Example Compounding Continuously

Suppose Paul invests \$400 at 8% annual interest compounded continuously. Find the value of his investment after 5 years.

$$P = 400, r = 0.08, \text{ and } t = 5,$$

$$A = Pe^{rt}$$

$$= 400e^{0.08(5)}$$

$$= 596.7298\dots$$

So Paul's investment is worth \$596.73.

# WP: Compound Interest

1. If \$100 is invested at 9% per annum compounded continuously, how long will it take before the amount is \$1000? Round the answer to two decimal places.

[A] 25.58 years      [B] 0.77 years      [C] 38.38 years      [D] 16.67 years

Turn and Talk:

1. What type of investment model should you use?
2. What information in the question helped you make your decision?

## WP: Compound Interest

2. Find the amount owed at the end of 5 years if \$3800 is loaned at a rate of 6% compounded semiannually.

[A] \$5714.11

[B] \$5106.88

[C] \$4281.43

[D] \$4198.95

### Turn and Talk:

1. What are you given?
2. What are you asked to find?
3. What is your model?

## WP: Compound Interest

3. What amount (to the nearest cent) will an account have after 10 years if \$50 is invested at 6.5% interest compounded continuously?

[A] \$95.78

[B] \$93.86

[C] \$95.28

[D] \$94.79

## WP: Compound Interest

4. How long does it take \$775 to double if it is invested at 7% interest compounded quarterly?
- [A] 12.3 months      [B] 10.0 months      [C] 12.3 years      [D] none of these

## What you'll learn about

- Annual Percentage Yield
- Annuities – Future Value
- Loans and Mortgages – Present Value

ADDITION FINANCE WORK

... and why

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## Annual Percentage Yield

A common basis for comparing investments is the **annual percentage yield (APY)** – the percentage rate that, compounded annually, would yield the same return as the given interest rate with the given compounding period.

## Example Computing Annual Percentage Yield

Meredith invests \$3000 with Frederick Bank at 4.65% annual interest compounded quarterly. What is the equivalent APY?

## Example Computing Annual Percentage Yield

Meredith invests \$3000 with Frederick Bank at 4.65% annual interest compounded quarterly. What is the equivalent APY?

Let  $x$  = the equivalent APY. The value after one year is  $A = 3000(1 + x)$ .

$$3000(1 + x) = 3000 \left( 1 + \frac{0.0465}{4} \right)^4$$

$$(1 + x) = \left( 1 + \frac{0.0465}{4} \right)^4$$

$$x = \left( 1 + \frac{0.0465}{4} \right)^4 - 1 = 0.047317\dots$$

The annual percentage yield is 4.73%.

## Example Computing Annual Percentage Yield

Jiamin borrows \$8000 with US Bank at 5.75% annual interest compounded daily. What is the equivalent APY?

## Annuities – Future Value

- It makes good financial sense to put smaller amounts of money into an account from each pay check rather than making a big deposit once a year.
- The previous investment models were for big lump sum investments or loans.

An **annuity** is a **sequence** of equal **periodic** payments. The annuity is **ordinary** if deposits are made at the end of the period at the same time as the interest is **posted** in the account.

## Future Value of an Annuity

The future value  $FV$  of an annuity consisting of  $n$  equal periodic payments of  $R$  dollars at an interest rate  $i$  per compounding period (payment interval) is

$$FV = R \frac{(1 + i)^n - 1}{i}.$$

## An Investment Annuity

Jolinda contributes to the Celebrity Retirement Fund that earns 10.4% annual interest. What should her monthly payments be if she wants to accumulate \$200,000 in 35 years.

## An Investment Annuity

Diego contributes to the Commercial National money market account that earns 4.25% annual interest. What should his monthly payments be if he wants to accumulate \$50,000 in 10 years.



## Why study Loans and Mortgages?

- Understand banking or be taken advantage of by commissioned salespeople.
- Mortgage brokers, bank account managers, and credit card companies don't have your best interest in mind; they want to sell something.
- They earn by selling you more than you need.

What are all of these finance professionals selling you?

## Present Value of an Annuity

The present value  $PV$  of an annuity consisting of  $n$  equal payments of  $R$  dollars at an interest rate  $i$  per period (payment interval) is

$$PV = R \frac{1 - (1 + i)^{-n}}{i}.$$

## House Mortgage Payment

Jane obtains a 25-year \$115,000 house loan with an APR of 13.25% from National City Bank. What is her monthly payment?

## House Mortgage Payment

Leslie obtains a 30-year \$96,000 house loan with an APR of 6.75% from National City Bank. What is her monthly payment?

# Rational Functions Poster Quiz

- Get team poster supplies.
- Get team handout.
- Choose and sign up for team problem.
- Use handout to meet all requirements.

Problem	Gallery Walk East Side Hall Team Names	Gallery Walk West Side Hall Team Names
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