3.3

Logarithmic Functions and Their Graphs

Quick Review

Evaluate the expression without using a calculator.

1. 6^{-2}

2. $\dfrac{8^{11}}{2^{32}}$

3. 7^0

Rewrite as a base raised to a rational number exponent.

4. $\dfrac{1}{\sqrt{e^3}}$

5. $\sqrt[4]{10}$

Quick Review Solutions

Evaluate the expression without using a calculator.

1. 6^{-2} $\quad \dfrac{1}{36}$

2. $\dfrac{8^{11}}{2^{32}}$ $\quad 2$

3. 7^{0} $\quad 1$

Rewrite as a base raised to a rational number exponent.

4. $\dfrac{1}{\sqrt{e^{3}}}$ $\quad e^{-3/2}$

5. $\sqrt[4]{10}$ $\quad 10^{1/4}$

What you'll learn about

- Inverses of Exponential Functions
- Common Logarithms – Base 10
- Natural Logarithms – Base e
- Graphs of Logarithmic Functions
- Measuring Sound Using Decibels

… and why

Logarithmic functions are used in many applications, including the measurement of the relative intensity of sounds.

Today’s Objectives

CO: Use basic logarithmic properties to solve logarithmic equations.

- Success Criteria
  - Translate between exponential and logarithmic functions
  - Describe the inverse relationship between exponential and logarithmic functions.
  - Examine basic properties of log functions

LO: Use sentence frames to examine the relationships between exponential and logarithmic functions and describe the process of solving logarithmic equations.

- Vocabulary: inverse

Changing Between Logarithmic and Exponential Form

If $x > 0$ and $0 < b \neq 1$, then $y = \log_b(x)$ if and only if $b^y = x$.

For **exponential** functions:
The domain/input of exponential functions are exponents.

The range/outputs are a number.

For **logarithmic** functions:
The domain/input of exponential functions are a number.

The range/outputs are an exponent.

Example **Solving Simple Logarithmic Equations**

Solve the equation by changing it to exponential form.

$$\log x = \log_{10}(x) = 4$$

Two Important Things to remember about converting between the logarithmic and exponential forms:

1. The base is the base or *b* = *b*.
2. Logarithmic functions output exponents.

Log Equations to Exponential Form

1. Write the equation $\log_{32} 4 = \frac{2}{5}$ in exponential form.

[A] $32^{2/5} = 4$ [B] $\left(\frac{2}{5}\right)^{32} = 4$ [C] $4^{2/5} = 32$ [D] $4^{5/2} = 32$

LO: We know the base of the log corresponds to the base of the exponential function, so the base = _____. The output of a log function is an exponent, so we know the exponent is ______. The input of a log function is the number, so we know that corresponds to the output of the exponential function, which equals _____. Therefore, our equation is _____________.

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Log Equations to Exponential Form

2. Write the equation $h = \log_{93} g$ in exponential form.

[A] $93 = h^g$ [B] $93^h = g$ [C] $93 = g^h$ [D] $93^g = h$

LO: We know the _______ of the log corresponds to the base of the exponential function, so the base = _____. The _________ of a log function is an exponent, so we know the exponent is _______. The _________ of a log function is the _________, so we know that corresponds to the ________ of the exponential function, which equals _____. Therefore, our equation is _______________.

4

Log Equations to Exponential Form

3. Write the equation $\log_{64} 16 = \frac{2}{3}$ in exponential form.

LO: We know the _______ of the log corresponds to the base of the exponential function, so the base = _____. The _________ of a log function is an exponent, so we know the exponent is _______. The _________ of a log function is the _________, so we know that corresponds to the ________ of the exponential function, which equals _____. Therefore, our equation is _______________.

4

Log Equations to Exponential Form

4. Write the equation $y = \log_{37} x$ in exponential form.

LO: We know the _______ of the log corresponds to the base of the exponential function, so the base = _____. The _________ of a log function is an exponent, so we know the exponent is _______. The _________ of a log function is the _________, so we know that corresponds to the ________ of the exponential function, which equals _____. Therefore, our equation is _______________.

4

AM: Exponential Equation to Log Form

1. Write the equation $6^2 = 36$ in logarithmic form.

[A] $\log_6 36 = 2$ [B] $\log_{\frac{1}{2}} 36 = 6$ [C] $\log_2 36 = 6$ [D] $\log_{36} 6 = 2$

LO: We know the base of the exponential function corresponds to the base of the log function, so the base = _____. The _________ of a log function is an exponent, so we know the exponent is ______. The _________ of an exponential function is the _________, so we know that corresponds to the ________ of the log function, which equals _____. Therefore, our equation is _____________.

5

AM: Exponential Equation to Log Form

2. Write the equation $16^{3/2} = 64$ in logarithmic form.

[A] $\log_{16} 64 = \frac{3}{2}$ [B] $\log_{3/2} 64 = 16$ [C] $2\log_3 64 = 16$ [D] $\log_{64} 16 = \frac{2}{3}$

LO: We know the base of the exponential function corresponds to the base of the log function, so the base = _____. The ________ of a log function is an exponent, so we know the exponent is ______. The ________ of an exponential function is the ________, so we know that corresponds to the _______ of the log function, which equals _____. Therefore, our equation is _____________.

5

AM: Exponential Equation to Log Form

3. Write the equation $4^3 = 64$ in logarithmic form.

LO: We know the base of the exponential function corresponds to the base of the log function, so the base = _____. The ________ of a log function is an exponent, so we know the exponent is ______. The ________ of an exponential function is the ________, so we know that corresponds to the _______ of the log function, which equals _____. Therefore, our equation is _____________.

5

AM: Exponential Equation to Log Form

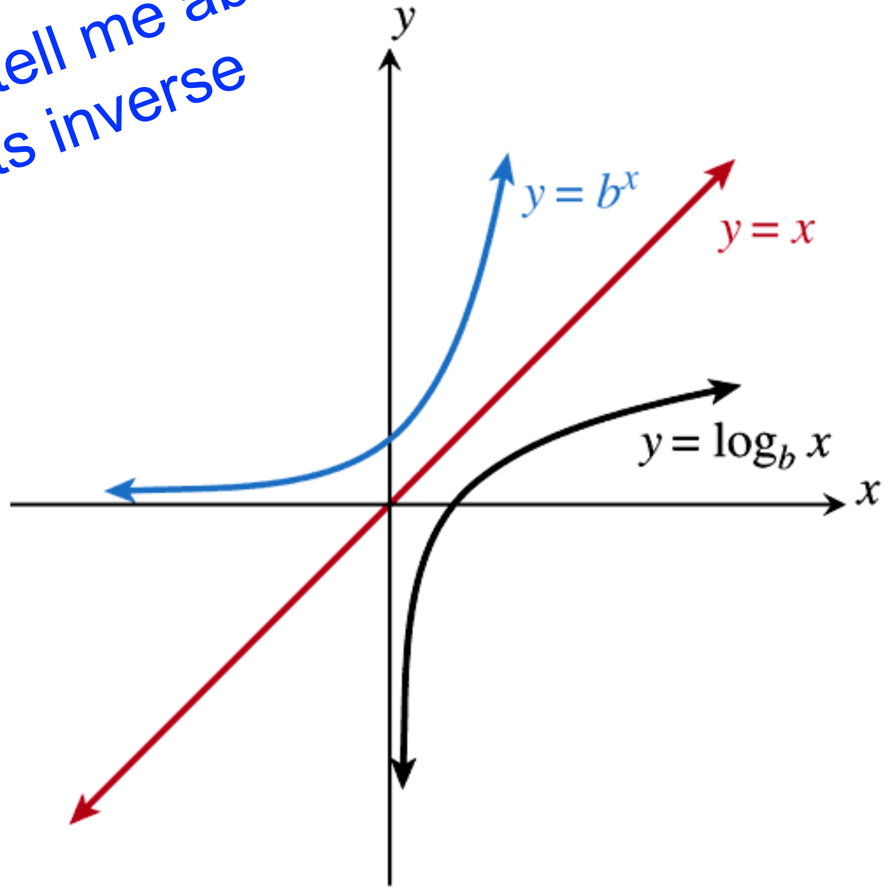
4. Write the equation $16^{3/4} = 8$ in logarithmic form.

LO: We know the base of the exponential function corresponds to the base of the log function, so the base = _____. The ________ of a log function is an exponent, so we know the exponent is ______. The ________ of an exponential function is the ________, so we know that corresponds to the _______ of the log function, which equals _____. Therefore, our equation is _____________.

5

Inverses of Exponential Functions

Turn and Talk:
What can you tell me about a function and its inverse function?



Quick Review: Functions and Inverses

LO:

•The____________ of the function is the __________________ of the __________________.

•The____________ of the function is the __________________ of the __________________.

•The graphs of a functions and its inverse are____________________over________________.

Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y.

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

An Exponential Function and Its Inverse

x	$f(x) = 2^x$
-3	$1/8$
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
3	8

x	$f^{-1}(x) = \log_2 x$
$1/8$	-3
$1/4$	-2
$1/2$	-1
1	0
2	1
4	2
8	3

LO:

- The___________ of the function is the ________________ of the ________________.

- The___________ of the function is the ________________ of the ________________.

Common Logarithm – Base 10

- Logarithms with base 10 are called common logarithms.
- The common logarithm $\log_{10}x = \log x$.
- The common logarithm is the inverse of the exponential function $y = 10^x$.

Basic Properties of Common Logarithms

Let x and y be real numbers with $x > 0$.

- $\log 1 = 0$ because $10^0 = 1$.
- $\log 10 = 1$ because $10^1 = 10$.
- $\log 10^y = y$ because $10^y = 10^y$.
- $10^{\log x} = x$ because $\log x = \log x$.

Example Solving Simple Logarithmic Equations

Solve the equation by changing it to exponential form.

$\log x = 4$

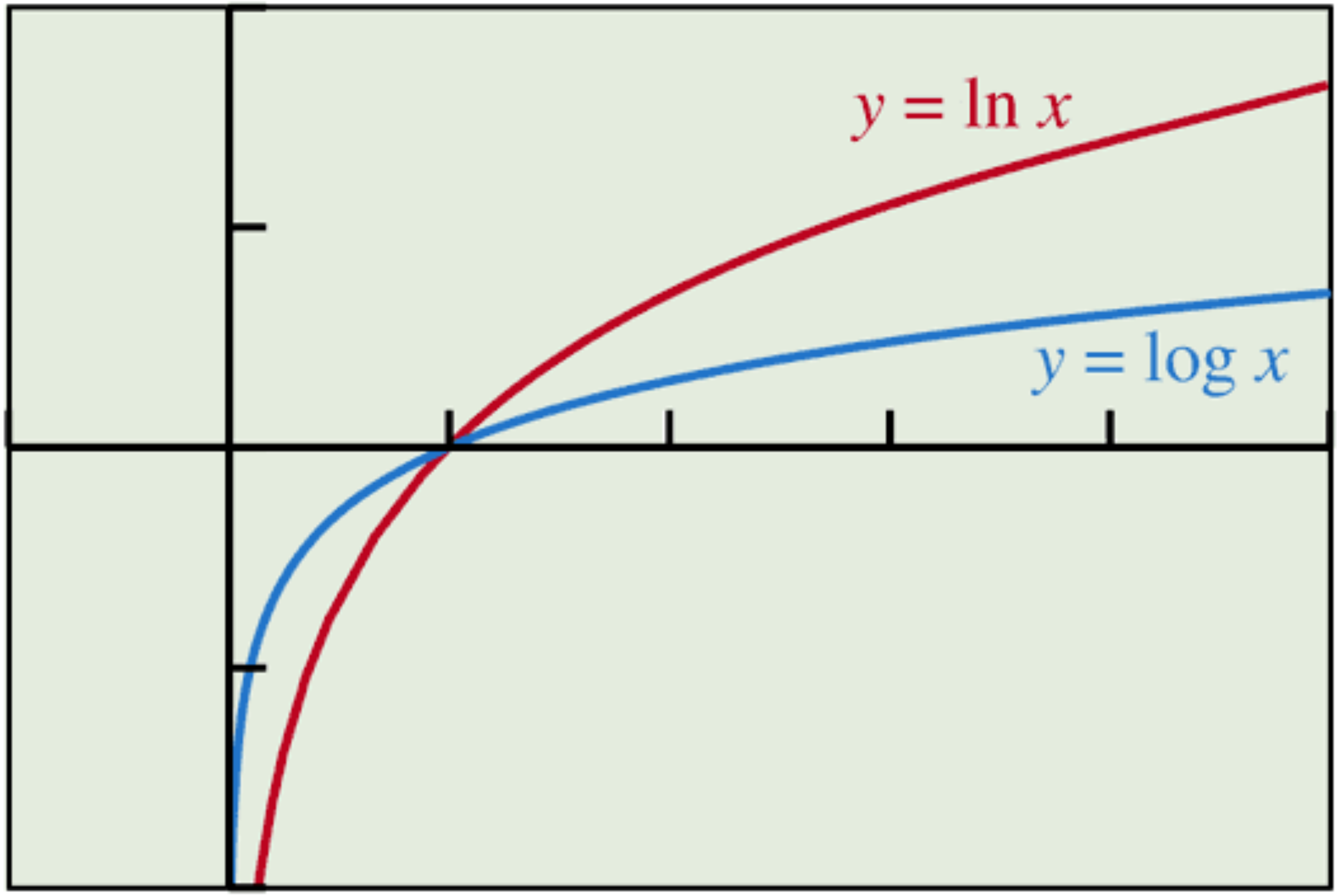
LO: We know the _______ of the log corresponds to the base of the exponential function, so the base = ______. The _________ of a log function is an exponent, so we know the exponent is _______. The _________ of a log function is the _________, so we know that corresponds to the ________ of the exponential function, which equals ______. Therefore, our equation is _______________.

Basic Properties of Natural Logarithms

Let x and y be real numbers with $x > 0$.

- $\ln 1 = 0$ because $e^0 = 1$.
- $\ln e = 1$ because $e^1 = e$.
- $\ln e^y = y$ because $e^y = e^y$.
- $e^{\ln x} = x$ because $\ln x = \ln x$.

Graphs of the Common and Natural Logarithm



[–1, 5] by [–2, 2]

Transforming Logarithmic Graphs

Describe how to transform the graph of $y = \ln x$ into the graph of $h(x) = \ln(2 - x)$.

Transforming Logarithmic Graphs

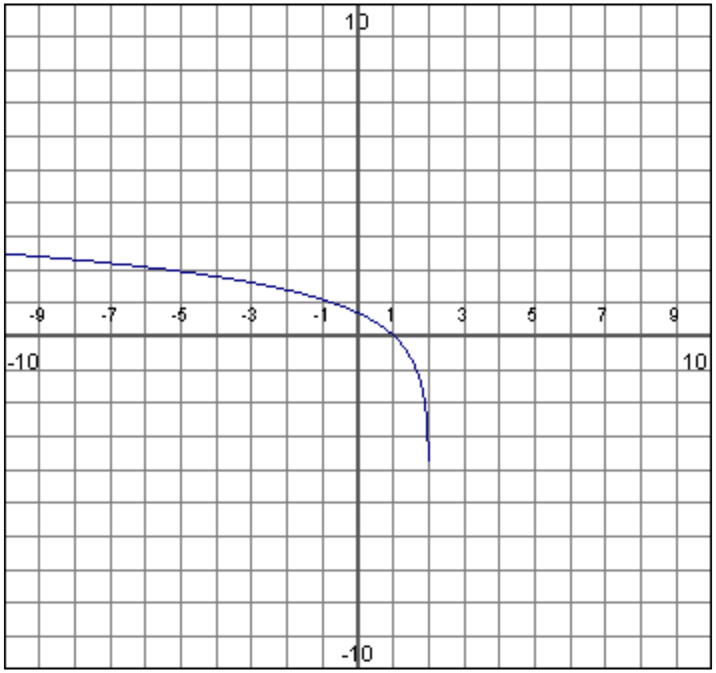
Describe how to transform the graph of $y = \ln x$ into the graph of $h(x) = \ln(2 - x)$.

$h(x) = \ln(2 - x) = \ln[-(x - 2)]$.

So obtain the graph of $h(x) = \ln(2 - x)$ from $y = \ln x$ by applying, in order,

a reflection across the y-axis followed by

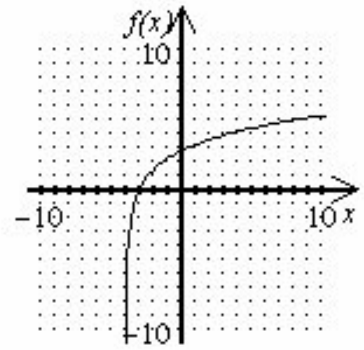
a translation 2 units to the right.



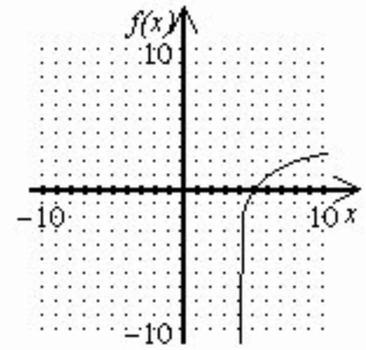
Graph Log Functions

1. Graph: $f(x) = \log_{1/2}(x-4)$

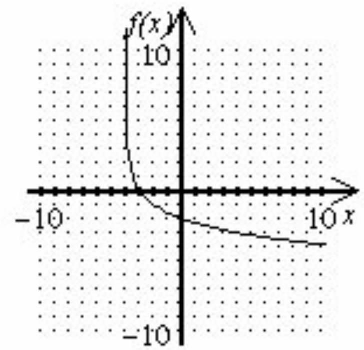
[A]



[B]



[C]



[D] none of these

LO: I know by looking at the equation that I have an a value of _______, and a b value of ________. I also have a shift of _____ units _______. Therefore, my answer is _______.

6

Decibels

The level of sound intensity in **decibels** (dB) is

$\beta = 10\log\left(\frac{I}{I_0}\right)$, where β (beta) is the number of decibels,

I is the sound intensity in W/m^2, and $I_0 = 10^{-12}\ \text{W/m}^2$ is the threshold of human hearing (the quietest audible sound intensity).

3.4

Properties of Logarithmic Functions

Quick Review

Evaluate the expression without using a calculator.

1. $\log 10^3$
2. $\ln e^3$
3. $\log 10^{-2}$

Simplify the expression.

4. $\dfrac{x^3 y^{-3}}{x^{-2} y^2}$
5. $\dfrac{\left(x^2 y^4\right)^{1/2}}{2x^{-3}}$

Quick Review Solutions

Evaluate the expression without using a calculator.

1. $\log 10^3$ 3
2. $\ln e^3$ 3
3. $\log 10^{-2}$ -2

Simplify the expression.

4. $\dfrac{x^3 y^{-3}}{x^{-2} y^2}$ $\dfrac{x^5}{y^5}$
5. $\dfrac{\left(x^2 y^4\right)^{1/2}}{2x^{-3}}$ $\dfrac{x^4 y^2}{2}$

What you'll learn about

- Properties of Logarithms
- Change of Base
- Graphs of Logarithmic Functions with Base *b*
- Re-expressing Data

… and why

The applications of logarithms are based on their many special properties, so learn them well.

Today’s Objectives

CO: Prove the logarithmic properties and apply them to expand and condense logarithmic function expressions.

- Success Criteria
  - Define the product, quotient, and power rule
  - Use the change of base formula to evaluate logarithmic expressions

LO: Use sentence frames to describe the process of condensing and expanding logarithmic expressions.

- Vocabulary: Product rule, quotient rule, power rule

Properties of Logarithms

Let b, R, and S be positve

real numbers with $b \neq 1$, and c any real number.

- **Product rule**: $\log_b(RS) = \log_b R + \log_b S$
- **Quotient rule**: $\log_b\left(\dfrac{R}{S}\right) = \log_b R - \log_b S$
- **Power rule**: $\log_b(R)^c = c\log_b R$

Proving the Product Rule for Logarithms

Prove $\log_b(RS) = \log_b R + \log_b S.$

Proving the Product Rule for Logarithms

Prove $\log_b(RS) = \log_b R + \log_b S$.

Let $x = \log_b R$ and $y = \log_b S$.

The corresponding exponential statements

are $b^x = R$ and $b^y = S$.

Therefore,

$RS = b^x \cdot b^y$

$RS = b^{x+y}$

$\log_b(RS) = x + y$ change to logarithmic form

$\log_b(RS) = \log_b R + \log_b S$

Expanding the Logarithm of a Product

Assuming x is positive, use properties of logarithms to write $\log\left(3x^5\right)$ as a sum of logarithms or multiple logarithms.

Expanding the Logarithm of a Product

Assuming x is positive, use properties of logarithms to write $\log\left(3x^5\right)$ as a sum of logarithms or multiple logarithms.

$$\log\left(3x^5\right) = \log 3 + \log\left(x^5\right)$$

$$= \log 3 + 5\log x$$

Condensing a Logarithmic Expression

Assuming x is positive, write $3\ln x - \ln 2$ as a single logarithm.

Condensing a Logarithmic Expression

Assuming x is positive, write $3\ln x - \ln 2$ as a single logarithm.

$$3\ln x - \ln 2 = \ln x^3 - \ln 2$$

$$= \ln\left(\frac{x^3}{2}\right)$$

Change-of-Base Formula for Logarithms

For positive real numbers $a, b,$ and x with $a \neq 1$ and $b \neq 1$,

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Evaluating Logarithms by Changing the Base

Evaluate $\log_3 10$.

Evaluating Logarithms by Changing the Base

Evaluate $\log_3 10$.

$$\log_3 10 = \frac{\log_{10} 10}{\log_{10} 3} = \frac{\log 10}{\log 3} = \frac{1}{\log 3} \approx 2.096$$

Properties of Logs, Rewrite Expressions

1. Express in terms of logarithms of x, y, and z: $\log_a 2xy^4z^2$

[A] $\dfrac{\log_a 2+\log_a x+4\log_a y}{2\log_a z}$

[B] $2+\log_a x+4\log_a y-2\log_a z$

[C] $\log_a 2+\log_a x+4\log_a y-2\log_a z$

[D] none of these

LO: First, I can use the product rule to expand the multiplied terms into the equivalent added logarithms. This will give me the expression ______________________________________. Next, I can use the power rule to expand each of these terms with the equivalent coefficients. This will give me the expression ______________________________________.

7

Properties of Logs, Rewrite Expressions

2. Write as the logarithm of a single expression: $5\log_3 x - 6\log_3(x-5)$

[A] $30\log_3 \frac{x}{x-5}$ [B] $\log_3 \frac{x^5}{(x-5)^6}$ [C] $\log_3 x^5(x-5)^6$ [D] none of these

LO: First, I can use the power rule to rewrite the coefficients into the equivalent exponents. This will give me the expression

_____________________________.

Next, I can use the quotient rule to condense each of these terms in the equivalent division form. This will give me the expression

________________________________.

7

Properties of Logs, Rewrite Expressions

3. Which is the logarithm written as a single expression?

$\log_a 3x + 3(\log_a x - \log_a y)$

[A] $\log_a \dfrac{3x^4}{y^3}$ [B] $\log_a \dfrac{6x}{3y}$ [C] $\log_a \dfrac{9x^2}{y}$ [D] none of these

LO: First, I can use the quotient rule to condense the subtracted terms into the equivalent division form. This will give me the expression ______________________.
Next, I can use the power rule to rewrite each of these terms with the equivalent coefficients. This will give me the expression _____________________________.
Lastly, I can use the product rule to condense the added terms in the equivalent multiplied form. This will give me the expression ____________________.

7

Properties of Logs, Rewrite Expressions

4. Write as the logarithm of a single expression: $\log_{11} 2 + \log_{11}(x+4) + \log_{11}(y+5)$

LO: I can use the __________ rule to condense the added terms into the equivalent multiplied form. This will give me the expression

____________________________________.

7

Turn and Talk: What's the point of properties of logarithms?

- Looking Back: Come up with four methods for solving quadratic equations?
- Discuss why you think these solution methods are important in your group and be ready to report back to the class.
- Now make a conjecture about why log properties are important and how you might use them.