Today's Objective

- Read an inequality problem and choose appropriate interval solutions **for polynomial and rational inequalities using** sign charts and graphical methods.
- Success Criteria
  - Set up appropriate inequalities for determining where an expression is positive or negative.
  - Construct and use a sign chart to solve inequalities involving polynomials
  - Construct and use a sign chart to solve inequalities involving rational expressions

Polynomial Inequalities

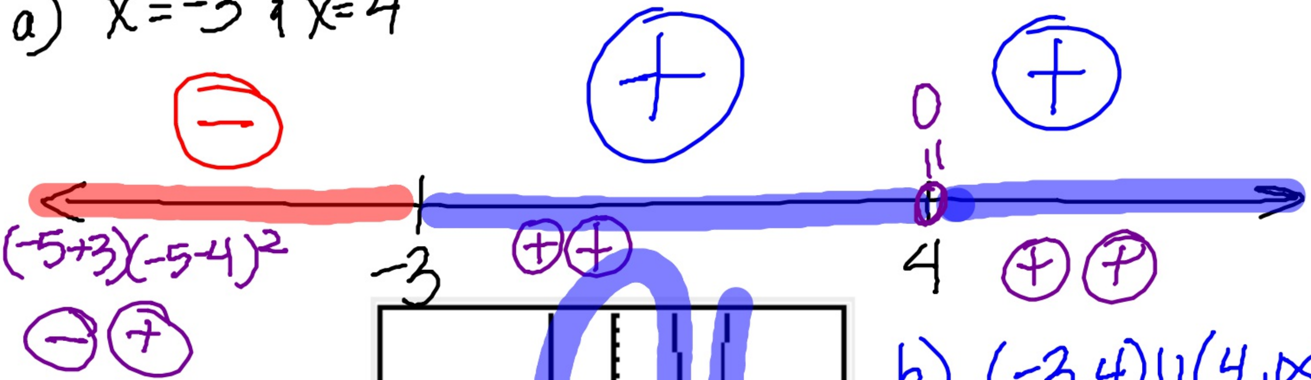
A polynomial inequality takes the form $f(x) > 0, f(x) \ge 0, f(x) < 0,$ $f(x) \le 0$ or $f(x) \ne 0$, where $f(x)$ is a polynomial.

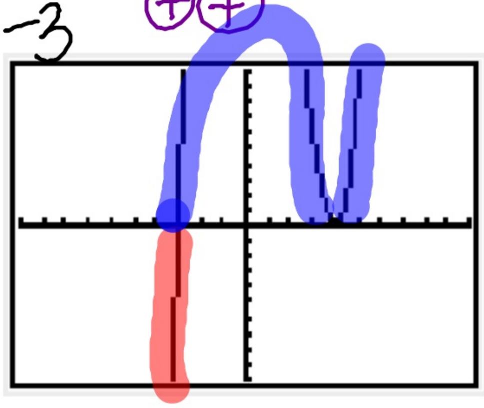
- To solve $f(x) > 0$ is to find the values of x that make $f(x)$ positive.
- To solve $f(x) < 0$ is to find the values of x that make $f(x)$ negative.

Example **Finding where a Polynomial is Zero, Positive, or Negative**

Let $f(x) = (x+3)(x-4)^2$. Determine the real number values of x that cause $f(x)$ to be (a) zero, (b) positive, (c) negative.

a) $x = -3$ & $x = 4$





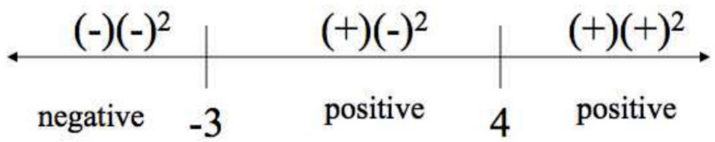
b) $(-3,4) \cup (4,\infty)$

c) $(-\infty,-3)$

Example **Finding where a Polynomial is Zero, Positive, or Negative**

Let $f(x) = (x+3)(x-4)^2$. Determine the real number values of x that cause $f(x)$ to be (a) zero, (b) positive, (c) negative.

(a) The real zeros are at $x = -3$ and at $x = 4$ (multiplicity 2).
Use a sign chart to find the intervals when $f(x) > 0$ and $f(x) < 0$.



(b) $f(x) > 0$ on the interval $(-3,4) \cup (4,\infty)$.
(c) $f(x) < 0$ on the interval $(-\infty,-3)$.

Example **Solving a Polynomial Inequality Graphically**

Solve $x^3 - 6x^2 \le 2 - 8x$ graphically.

$+8x - 2 \quad -2 + 8x$

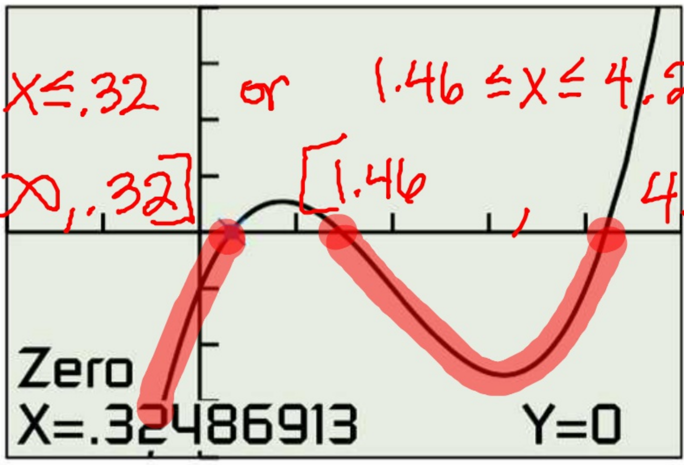
$$\underbrace{x^3 - 6x^2 + 8x - 2}_{y_1} \le 0$$

Example **Solving a Polynomial Inequality Graphically**

Solve $x^3 - 6x^2 \le 2 - 8x$ graphically.

Rewrite the inequality: $x^3 - 6x^2 + 8x - 2 \le 0$ ← 0 or Negative

Find the zeros of the function graphically:



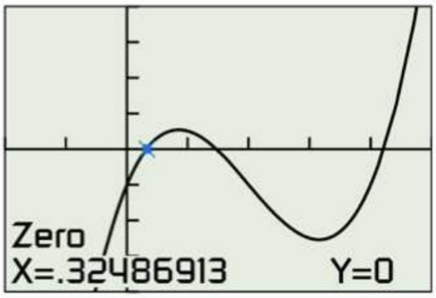
[–2, 5] by [–8, 8]

Example **Solving a Polynomial Inequality Graphically**

Solve $x^3 - 6x^2 \le 2 - 8x$ graphically.

Rewrite the inequality $x^3 - 6x^2 + 8x - 2 \le 0$.

Let $f(x) = x^3 - 6x^2 + 8x - 2$ and find the real zeros of f graphically.



[–2, 5] by [–8, 8]

The three real zeros are approximately 0.32, 1.46, and 4.21. The solution consists of the x values for which the graph is on or below the x-axis. The solution is $(-\infty, 0.32] \cup [1.46, 4.21]$.

Example **Creating a Sign Chart for a Rational Function**

Let $r(x) = \dfrac{x+1}{(x+3)(x-1)}$. Determine the values of x that cause $r(x)$ to be

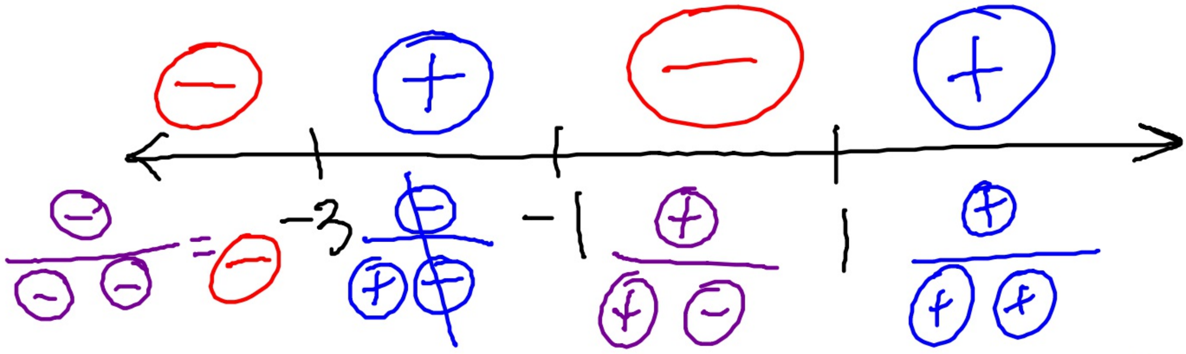
(a) zero, (b) undefined, (c) positive, and (d) negative.

a) $x = -1$

b) $x = -3 \quad x = 1$

c) $(-3, -1) \cup (1, \infty)$

d) $(-\infty, -3) \cup (-1, 1)$



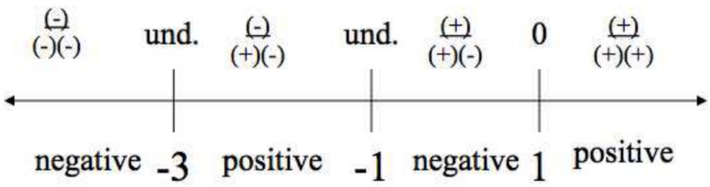
Example **Creating a Sign Chart for a Rational Function**

Let $r(x) = \dfrac{x+1}{(x+3)(x-1)}$. Determine the values of x that cause $r(x)$ to be

(a) zero, (b) undefined, (c) positive, and (d) negative.

(a) $r(x) = 0$ when $x = -1$.

(b) $r(x)$ is undefined when $x = -3$ and $x = 1$.



(c) $(-3,-1) \cup (1,\infty)$

(d) $(-\infty,-3) \cup (-1,1)$

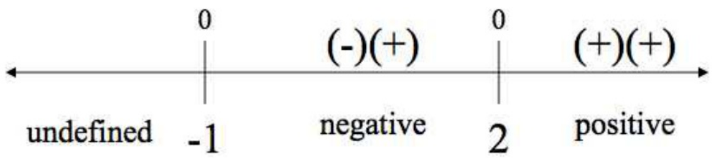
Example **Solving an Inequality Involving a Radical**

Solve $(x-2)\sqrt{x+1} \le 0$.

Example **Solving an Inequality Involving a Radical**

Solve $(x-2)\sqrt{x+1} \le 0$.

Let $f(x) = (x-2)\sqrt{x+1}$. Because of the factor $\sqrt{x+1}$, $f(x)$ is undefined if $x < -1$. The zeros are at $x = -1$ and $x = 2$.



$f(x) \le 0$ over the interval $[-1,2]$.

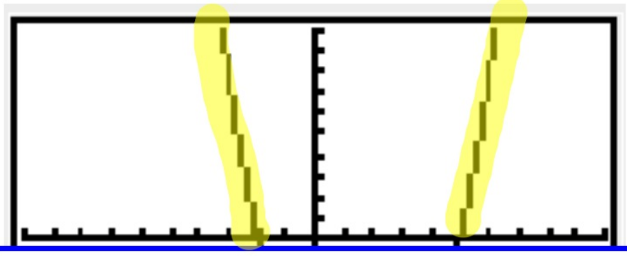
AM: Solve Polynomial Inequalities

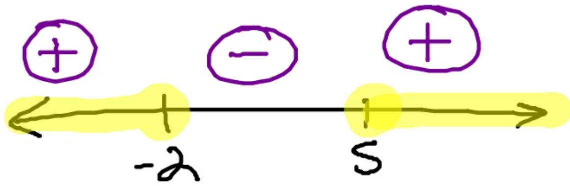
1. Identify the solution set for the inequality: $x^2 - 3x \geq 10$

[A] $-5 \leq x \leq 2$ [B] $x \leq -5$ or $x \geq 2$ [C] $-2 \leq x \leq 5$ ([D]) none of these

$x^2 - 3x - 10 \geq 0$

$(x-5)(x+2) \geq 0$





$x \leq -2$ OR $x \geq 5$

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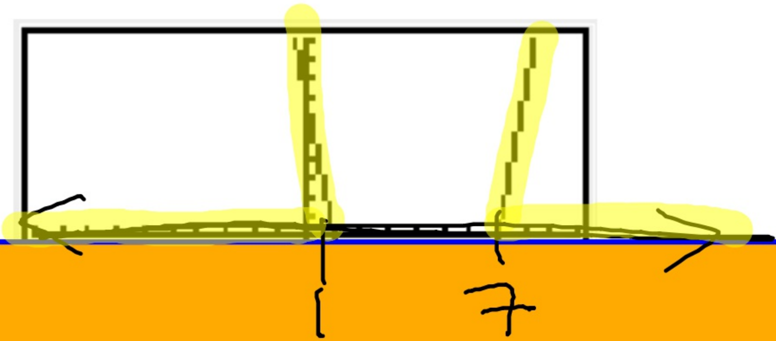
AM: Solve Polynomial Inequalities

2. Solve: $x^2 - 8x + 7 > 0$

[A] $-7 < x < -1$ [B] $x < 1$ or $x > 7$ [C] $x < -7$ or $x > -1$ [D] $1 < x < 7$



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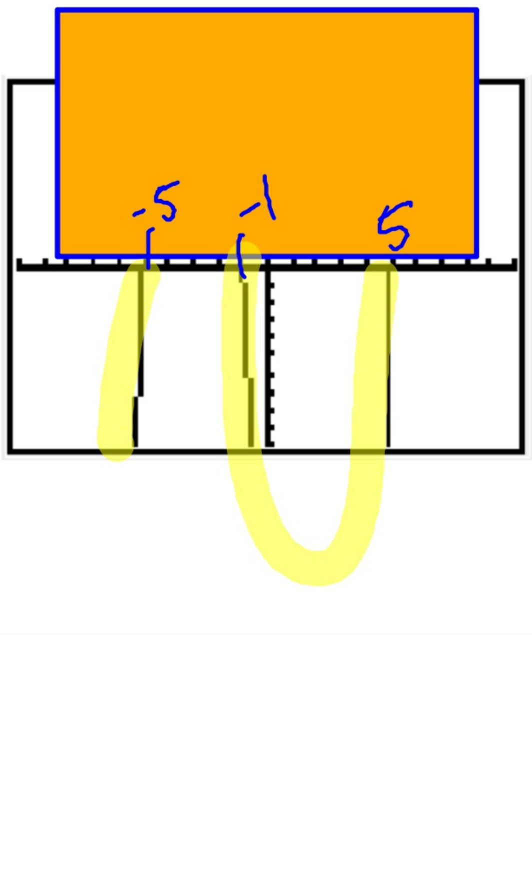
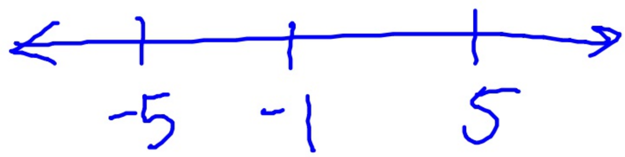
AM: Solve Polynomial Inequalities

3. Identify the solution for the inequality: $x^3 + x^2 - 25x - 25 < 0$

[A] $(-\infty, -5) \cup (-1, 1)$ [B] $\left(-2, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

[C] $(-\infty, -5) \cup (-1, 5)$ [D] $\left(-\frac{3}{2}, 1\right) \cup \left(\frac{3}{2}, \infty\right)$





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AM: Solve Polynomial Inequalities

4. Solve: $x^2 - 7x \geq 18$

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AM: Solve Inequalities with Rational Expressions

1. Solve: $\frac{7x+1}{x-1} \le 9$

[A] $x < 1$ or $x \ge 5$ [B] $x < 5$ or $x \ge 1$ [C] $x \le 1$ or $x > 5$ [D] $1 < x \le 5$

$$\frac{7x+1}{x-1} \le 9$$

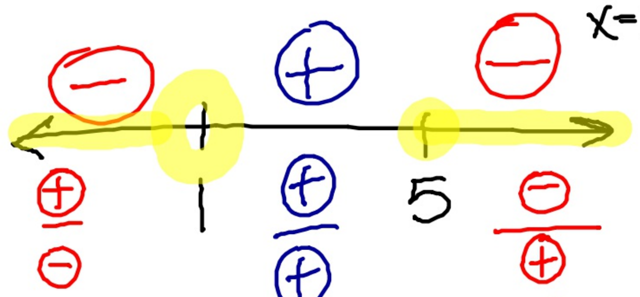
$$-9 \qquad -9$$

$$\frac{7x+1}{x-1} - 9\frac{(x-1)}{(x-1)} \le 0$$

$$\frac{7x+1-9x+9}{x-1} \le 0$$

$$\frac{-2x+10}{x-1} \le 0$$

$$-2x+10=0$$
$$-10 \quad -10$$
$$\frac{-2x}{-2} = \frac{-10}{-2}$$
$$x=5$$



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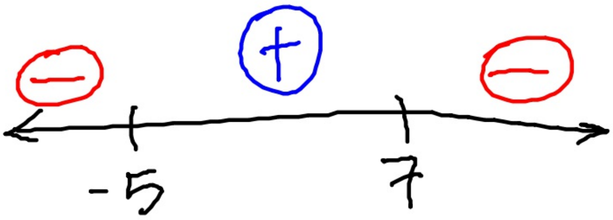
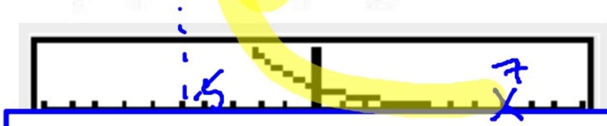
AM: Solve Inequalities with Rational Expressions

2. Find the solution set in interval notation: $\frac{x+41}{x+5} \geq 4$

[A] $(-\infty, -5) \cup [41, \infty)$ [B] $(-5, 41]$ [C] $[7, \infty)$ [D] $(-5, 7]$

$$\frac{-3x+21}{x+5} \geq 0$$





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AM: Solve Inequalities with Rational Expressions

Solve:

3. $\frac{21}{20} - \frac{1}{5}x + \frac{19}{20} \geq 5x - \frac{6}{5} \cdot 20$

[A] $x \leq \frac{8}{13}$ [B] $x \leq \frac{2}{3}$ [C] $x \geq \frac{8}{13}$ [D] none of these

$21 - 4x + 19 \geq 100x - 24$

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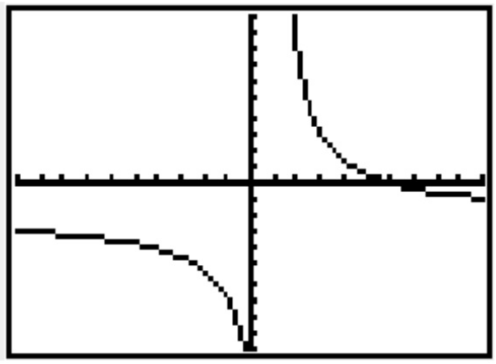
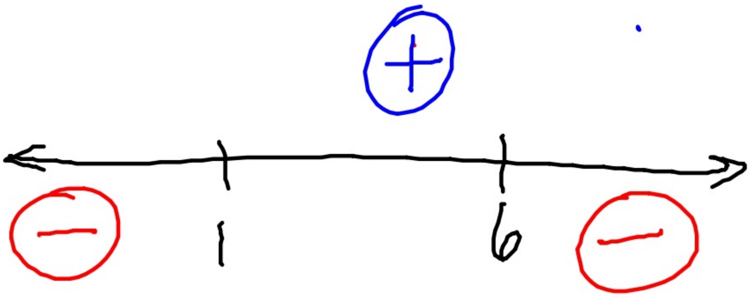
AM: Solve Inequalities with Rational Expressions

4. $\frac{9x+1}{x-1} \leq 11$

$\frac{-2x+12}{x-1} \leq 0$

⊖ or 0 $(-\infty, 1) \cup [6, \infty)$





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AM: Solve Inequalities with Rational Expressions

1. Solve: $\dfrac{(x-5)(x+3)}{x-3} \le 0$

[A] $x \le -3$ or $3 < x \le 5$

[B] $3 \le x \le -5$

[C] $x \le -3$ or $x \ge -5$

[D] $x \ge 5$ or $-3 \le x < 3$

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AM: Solve Inequalities with Rational Expressions

2. Identify the solution set of the inequality: $\dfrac{7x+1}{x-1} \le 9$

[A] $x \ge 5$ [B] $1 < x \le 5$ [C] $x < 1$ or $x \ge 5$ [D] $1 \le x \le 5$

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AM: Solve Inequalities with Rational Expressions

3. $\dfrac{(x-7)(x+4)}{x-3} \geq 0$

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AM: Solve Inequalities with Rational Expressions

4. $\dfrac{x+1}{x-1} \leq 3$

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