Rate of Work Problems

- The amount of work that you do depends on the rate at which you work and the amount of time that you spend working.

Work = Rate · Time

w : The amount of work completed.

r : The rate at which work is completed.

t : The amount of time spent working.

$w = r \cdot t$

- Suppose two people are working together to complete the same job. One employee is faster then the other, but they will both work until the job is done. Common sense tells us that the combined rate will be faster then the fastest worker, but what exactly is the combined rate?

AM: WP: Rational Expressions

1. Janet can paint a kitchen in 3 hours and Roger can paint the same kitchen in 5 hours. How long would it take for both working together to paint the kitchen?

[A] $1\frac{7}{8}$ hr [B] 8 hr [C] $\frac{8}{15}$ hr [D] 4 hr

Things to consider when solving rate of work problems

- Does each person complete the same amount work?
- Does each person work the same amount of time?
- How long does it take them to complete the job together?
- Who is faster? Who is slower?
- How fast can each person do the work independently?
- How fast can they work as team?
- ***Does your solution make sense?***

$W = R \cdot T$

$\frac{W}{T} = R$

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1. Clearly define the variables, state the units and give appropriate restrictions.

R_j : Janet's rate

R_r : Roger's rate

R_c : Combined rate

W_c : Combined Work

T_c : Combined time

Janet	Roger
$T_j = 3$ hrs.	$T_r = 5$ hrs
$W_j = 1$ kitchen	$W_r = 1$ kitchen
$R_j = \frac{1 \text{ kitchen}}{3 \text{ hrs}}$	$R_r = \frac{1 \text{ kitchen}}{5 \text{ hrs.}}$

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2. Write a mathematically correct equations that connects all *relevant* mathematical information.

$$R_c = R_j + R_r$$

$$W_c = R_c \cdot T_c$$

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3. Solve the equation with an appropriate algebraic method and confirm your solution with a graph.

$$R_c = R_j + R_r$$

$$R_c = \frac{1}{3}\frac{kitchen}{hours} + \frac{1}{5}\frac{kitchen}{hours} = \frac{1}{3}\frac{5}{\cdot 5}\frac{kitchen}{hours} + \frac{1}{5}\frac{\cdot 3}{\cdot 3}\frac{kitchen}{hours}$$

$$R_c = \frac{8}{15}\frac{kitchen}{hours}$$

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TURN AND TALK: Do a common sense check. If alone Janet, the faster worker, can paint a 1/3 of a kitchen in 1 hour. We determined that together they can paint 8/15 of a kitchen in one hour. Explain why this answer makes sense.

$$R_c = R_j + R_r$$

$$R_c = \frac{1}{3}\frac{kitchen}{hours} + \frac{1}{5}\frac{kitchen}{hours} = \frac{1}{3}\frac{5}{5}\frac{kitchen}{hours} + \frac{1}{5}\frac{3}{3}\frac{kitchen}{hours}$$

$$R_c = \frac{8}{15}\frac{kitchen}{hours}$$

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4. Solve the equation with an appropriate algebraic method and confirm your solution with a graph.

$$W_c = R_c \cdot T_c$$

$$W_c = \frac{8 \text{ kitchen}}{15 \text{ hours}} \cdot T_c$$

$$\frac{15 \text{ hrs}}{8 \text{ kitchen}} 1 \text{ kitchen} = \frac{8 \text{ kitchen}}{15 \text{ hours}} \cdot T_c \cdot \frac{15 \text{ hours}}{8 \text{ kitchen}}$$

$$T_c = 1\frac{7}{8} \text{ hrs.}$$

AM: WP: Rational Expressions

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5. State your answer in a complete sentence, which conveys the real world meaning of the work rate problem solution.

LO: Janet's work rate is R_j =_____. Roger's work rate is R_r =_____. The combined work rate is R_C =_____, therefore it will take them _____ hours to paint _____ kitchen.

AM: WP: Rational Expressions

2. Working alone, Dale can paint a certain room in 6 hours. Jan, working alone, can paint the same room in 9 hours. How long will it take Dale and Jan to paint the room if they work together?

[A] $7\frac{1}{2}$ hr [B] $3\frac{7}{16}$ hr [C] $3\frac{3}{4}$ hr [D] $3\frac{3}{5}$ hr

LO: _____ work rate is _____ =_____.
_____ work rate is _____ =_____. The combined work rate is R_C =_____, therefore it will take them _____ hours to paint _____.

AM: WP: Rational Expressions

3. Find two consecutive even integers such that the sum of their reciprocals is $\frac{9}{40}$.

[A] 8 and 10 [B] 8 and 9 [C] 6 and 8 [D] 10 and 12

$x =$ 1st integer Reciprocal $= \frac{1}{x}$

$x+2 =$ 2nd integer Reciprocal $= \frac{1}{x+2}$

LCD: $40x(x+2)$

$$40x(x+2)\frac{1}{x} + \frac{1}{x+2}40x(x+2) = \frac{9}{40}40x(x+2)$$

$$40x+80+40x = 9x^2+18x$$

$$80x+80 = 9x^2+18x$$

$$-80x-80 \qquad -80x-80$$

$$0 = 9x^2-62x-80$$

$x = 8$

AM: WP: Rational Expressions

4. Together, Hatsumi and John can write a particular type of computer program in 17 hours. Alone, Hatsumi can do the job 4 hours faster than John. Find the time that each person takes to write a computer program.

[A] Hatsumi: 1.9 hours, John: 5.9 hours

[B] Hatsumi: 30.9 hours, John: 26.9 hours

[C] Hatsumi: 17 hours, John: 13 hours

[D] none of these

1. Clearly define the variables, state the units and give appropriate restrictions.

R_j : John's Rate

R_h : Hatsumi's Rate

R_c : Combined Rate

W_c : Combined Work

T_j : John's Time

T_h : Hatsumi's Time

AM: WP: Rational Expressions

4. Together, Hatsumi and John can write a particular type of computer program in 17 hours. Alone, Hatsumi can do the job 4 hours faster than John. Find the time that each person takes to write a computer program.

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[B] Hatsumi: 30.9 hours, John: 26.9 hours

[C] Hatsumi: 17 hours, John: 13 hours

[D] none of these

2. Write a mathematically correct equations that connects all *relevant* mathematical information.

$$R_j = \frac{1}{T_j}\frac{cp}{hours}$$

$$R_h = \frac{1}{T_h}\frac{cp}{hours} = \frac{1}{T_j - 4}\frac{cp}{hours}$$

$$R_c = \frac{1}{17}\frac{cp}{hours}$$

$$R_c = R_j + R_h$$

so $T_h = T_j - 4$

$$\frac{1\,cp}{17\,hrs} = \frac{1\,cp}{T_j\,hrs} + \frac{1\,cp}{T_j - 4\,hrs}$$

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2. Write a mathematically correct equations that connects all *relevant* mathematical information.

$$\frac{1}{17}\frac{cp}{hours} = \frac{1}{T_j}\frac{cp}{hours} + \frac{1}{T_j - 4}\frac{cp}{hours}$$

Let $T_j = x$

LCD: $17x(x-4)$

$$\frac{17x(x-4)1}{17} = \frac{1}{x}(17x(x-4)) + \frac{1}{x-4}17x(x-4)$$

$$x^2 - 4x = 17x - 68 + 17x$$

$$-x^2 + 4x \quad -x^2 + 4x$$

$$0 = -x^2 + 38x - 68$$