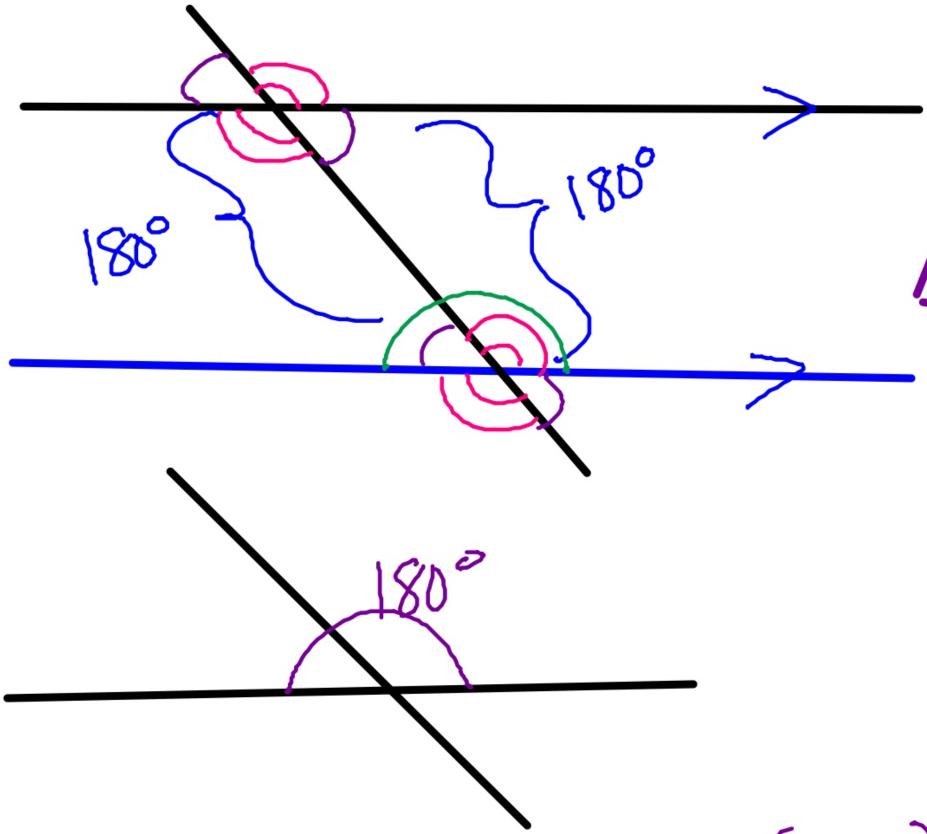
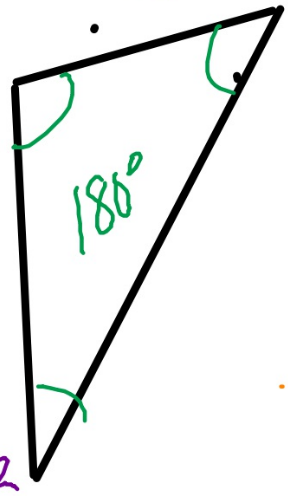
Today's Objectives

- **Perform mathematical operations with rational expressions to solve real-world problems** and write answers in complete sentences that describe the real-world meaning using sentence stems. Assess for **extraneous solutions**.

- Success Criteria:
  - Define extraneous solutions
  - Find the LCD for rational expressions
  - Examine LCD process for extraneous solutions

- **Vocabulary:** numerator, denominator, extraneous solution, least common denominator (LCD), greatest common factor (GCF), prime number, prime polynomial, irreducible quadratic, linear





$(x+7)^2$

$(x+7)(x+7)$

Extraneous Solutions

When we multiply or divide an equation by an expression containing variables, the resulting equation may have solutions that are *not* solutions of the original equation. These are **extraneous solutions**. For this reason we must check each solution of the resulting equation in the original equation.

LCD: x

Example **Solving by Clearing Fractions**

Solve $x + \frac{2}{x} = 3$.

$$x \cdot x + \frac{2}{\cancel{x}} \cdot \cancel{x} = 3 \cdot x$$

$$x^2 + 2 = 3x$$

$$-3x \quad -3x$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$x = 2$ or $x = 1$

LO: The least common denominator is needed to elimnate the fraction in this equation, therefore we must multiply both sides of the equation by the LCD. The new equation is a quadratic equation which can be solved by using the quadratic formula, ________________ rule, ________________ the square or a ______________.

Example **Solving by Clearing Fractions**

Solve $x + \frac{2}{x} = 3.$

The LCD is x.

$x + \frac{2}{x} = 3$

$x^2 + 2 = 3x$ multiply by x

$x^2 - 3x + 2 = 0$ subtract $3x$

$(x-2)(x-1) = 0$ factor

$x = 2$ or $x = 1$

Confirm algebraically:

Let $x = 2$: $2 + \frac{2}{2} = 3$

Let $x = 1$: $1 + \frac{2}{1} = 3$ Each value is a solution of the original equation.

LCD: $(x-3)(x-1)$

Example **Eliminating Extraneous Solutions**

Solve the equation $\dfrac{1}{x-3}+\dfrac{2x}{x-1}=\dfrac{2}{x^2-4x+3}$.

$3 \to 0$

$$\cancel{(x-3)}(x-1)\frac{1}{\cancel{x-3}}+\frac{2x}{\cancel{x-1}}(x-3)\cancel{(x-1)}=\frac{2}{\cancel{(x-1)}\cancel{(x-3)}}\cancel{(x-3)}\cancel{(x-1)}$$

$$x-1+2x^2-6x=2$$

$$2x^2-5x-1=2$$

$$-2 \quad -2$$

$$2x^2-5x-3=0$$

$$(x-3)(2x+1)=0$$

$\cancel{x=3}$ or $x=-1/2$

$$\begin{array}{r|rrr} 3 & 2 & -5 & -3 \\ & +\downarrow & 6 & 3 \\ \hline & 2 & 1 & 0 \end{array}$$

$\underbrace{2 \quad 1}_{2x+1}$

A) -2 B) 0 C) 2 D) none of these

$$\frac{x}{x+2} - \frac{1}{3} = -\frac{2}{x+2}$$

-2 0

LCD: $3(x+2)$

$$3\cancel{(x+2)}\frac{x}{\cancel{x+2}} - \frac{1}{\cancel{3}}\cancel{3}\cdot(x+2) = \frac{-2}{\cancel{x+2}}3\cancel{(x+2)}$$

$$3x-(x+2)=-6$$

$$3x-x-2=-6$$

$$2x-2=-6$$
$$+2 \quad +2$$

$$\frac{2x}{2}=\frac{-4}{2}$$

$\rightarrow \cancel{x=-2}$

Example **Eliminating Extraneous Solutions**

Solve the equation $\dfrac{1}{x-3}+\dfrac{2x}{x-1}=\dfrac{2}{x^2-4x+3}$.

The LCD is $(x-1)(x-3)$.

$$(x-1)(x-3)\left(\frac{1}{x-3}+\frac{2x}{x-1}\right)=(x-1)(x-3)\left(\frac{2}{x^2-4x+3}\right)$$

$$(x-1)(1)+2x(x-3)=2$$

$$2x^2-5x-3=0$$

$$(2x+1)(x-3)=0$$

$$x=-1/2 \quad \text{or} \quad x=3$$

Check solutions in the original equation. $x=-1/2$ is the only solution. The original equation is not defined at $x=3$.

LCD: $(x+4)(x+9)$

AM: Solve Rational Equations

Solve:

1. $\dfrac{x+6}{x+4}=\dfrac{x+8}{x+9}$ [A] $\dfrac{86}{3}$ [B] $\dfrac{16}{27}$ [C] $-\dfrac{86}{15}$ [D] none of these

$$\cancel{(x+4)}(x+9)\frac{x+6}{\cancel{x+4}}=\frac{x+8}{\cancel{x+9}}(x+4)\cancel{(x+9)}$$

$$x^2+6x+9x+54=x^2+4x+8x+32$$

$$\cancel{x^2}+15x+54=\cancel{x^2}+12x+32$$
$$-\cancel{x^2} \qquad -\cancel{x^2}$$

$$15x+54=12x+32$$
$$-12x \qquad -12x$$

$$3x+54=32$$
$$-54 \quad -54$$

$$\frac{3x}{3}=\frac{-22}{3}$$

$$x=-\frac{22}{3}$$

AM: Solve Rational Equations

2. $\dfrac{x}{x-6}+\dfrac{5}{x-3}=\dfrac{x^2}{x^2-9x+18}$ [A] 19 ~~[B]~~ 6 [C] 15 [D] none of these

LO: The __________ __________ ______________ is needed to ____________ the fractions in this equation, therefore we must multiply both sides of the equation by the __________. The new equation is a _______________ equation which can be solved by ____.

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