$$\frac{-1-1}{(-1+3)(-1-2)}$$

$$f(x)=\frac{x^2-1}{\boxed{(x^2+x-6)}(x+1)}$$

$$f(x)=\frac{\cancel{(x+1)}(x-1)}{(x+3)(x-2)\cancel{(x+1)}}\begin{array}{l}\rightarrow -1,\ 1\\ \rightarrow -3,\ 2,\ -1\end{array}$$

-1	B	(x+1)	1/1			
1	N	(x-1)	1			
-3	D	(x+3)	1			
2	D	(x-2)	1			

5th 6 Weeks

13 Obj. → Zeros of Polynomials.

5 Obj → SB Factoring DT

1-4 Obj → Rational Functions

22 Obj.

x-int $(-2,0)$

$$f(x) = \frac{(x+10)^3 (x-7) (x+2)^2}{(x-7)^5 (x+10)(x-5)}$$

$\rightarrow -10, 7, -2$

$\rightarrow 7, -10, 5$

True or False

$$\frac{(x+10)^{2}}{\cancel{(x+10)}}$$

~~There is R.D. @ x = 7.~~

There is a V.A. @ x=7 x=5

$\boxed{\text{There is a R.D. @ } x=-10}$

$$f(x) = \frac{\cancel{(x-\pi)^3}(x-5)(x+7)^2}{(x+7)^4(x+27)^2\cancel{(x-\pi)^3}}$$

$\to \overset{3}{\pi},\ \overset{1}{5},\ \overset{2}{-7}$

$\to \underset{4}{-7},\ \underset{2}{-27},\ \underset{3}{\pi}$

$= ?$

End Behavior:

H.A. $y=0$

R.D. : $x = \pi$

V.A. : $x=-7;\ x=-27$

X-int: $(5,0)$

$=$

a. End Behavior

$$f(x)=\frac{x^2-1}{(x^2+x-6)(x+1)} \quad \frac{(x+1)(x-1)}{(x+3)(x-2)(x+1)}$$

LO: The degree of the numerator, *n*, is __2__ and the degree of the denominator, *m*, is __3__, because these are the __largest__ powers of *x* for the respective polynomial functions. Since __$n<m$__, we can conclude that the end behavior asymptote is the __horizontal asymptote__, __$y=0$__, that means as x approaches __$\pm\infty$__ y approaches __0__.

$$f(x) = \frac{100\,\cancel{(x+3)}^2\,\cancel{(x-12)}^3\,(x-3)^2}{25\,\cancel{(x+3)}^4\,\cancel{(x+13)}\,\cancel{(x-12)}^2} \rightarrow \begin{array}{l} -3^2, 12^3, 3^2 \\ -3^4, -13^1, 12^2 \end{array}$$

R.D.: $x=12 \rightarrow (12,0)$ o

V.A.: $x=-3 \quad x=-13$

x-int: $(3,0)$

End Behavior: H.A. @ $y=\frac{100}{25}$, $y=4$

2. **Vertical asymptotes, removable discontinuities and x-intercepts**:

   b. Removable discontinuities or holes occur when the numerator and denominator share common factors that can be canceled form the denominator

$$f(x)=\frac{x^2-1}{\left(x^2+x-6\right)\left(x+1\right)} \quad \frac{\boxed{\cancel{(x+1)}}(x-1)}{(x+3)(x-2)\boxed{\cancel{(x+1)}}}$$

R.D. @ $x=-1$ Since $(x+1)$ is a factor of both the numerator and denominator and cancels completely from the denominator.

$$\frac{(-1-1)}{(-1+3)(-1-2)}=\frac{-2}{-6}=\frac{1}{3}$$

hole @ $(-1, 1/3)$.

2. **Vertical asymptotes, removable discontinuities and x-intercepts**:

   c. The x-intercepts occur at the zeros of the numerator that are not holes.

$$f(x)=\frac{x^2-1}{(x^2+x-6)(x+1)} = \frac{\cancel{(x+1)}(x-1)}{(x+3)(x-2)\cancel{(x+1)}}$$

$(1, 0)$

2. **Vertical asymptotes, removable discontinuities and x-intercepts**:

d. The vertical asymptotes occur at the zeros of the denominator that are not holes.

$$f(x)=\frac{x^2-1}{(x^2+x-6)(x+1)} \quad \frac{\cancel{(x+1)}(x-1)}{(x+3)(x-2)\cancel{(x+1)}}$$

R.D.

X=-3 and x=2

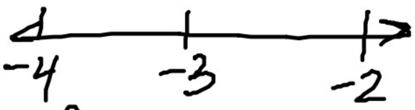
y-intercepts always have an x value of 0.

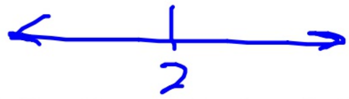
y-intercept: Find $f(0)$.

$$f(x)=\frac{x^2-1}{(x^2+x-6)(x+1)}$$

$$\frac{0^2-1}{(0^2+0-6)(0+1)}=\frac{1}{6}$$

$$\left(0,\frac{1}{6}\right)$$





f. Give numerical evidence to demonstrate the functions behavior at all vertical asymptotes.

$x = -3$

LEFT

x	f(x)
-3.999	-.8341
-3.73	-1.131
-3.2	-4.038
-3.001	-800

$-\infty$

RIGHT

x	f(x)
-2.5	1.5556
-2.7	2.6241
-2.89	7.2318
-2.99	79.96

-2.9999

∞

$x = 2$

LEFT

x	f(x)
1.5	-.2222
1.8	-.8333
1.99	-19.84
1.999	-199.8

$-\infty$

x	f(x)
2.9	.35782
2.6	.47619
2.2	1.1538
2.01	20.16

2.0001

∞

g. Use limit notation to write **left hand and right hand limits** that describes the functions behavior near each vertical asymptote, and express each analytical limit as sentence.

$$\lim_{x \to -3^-} f(x) = -\infty$$

$$\lim_{x \to -3^+} f(x) = \infty$$

$$\lim_{x \to 2^-} f(x) = -\infty$$

$$\lim_{x \to 2^+} f(x) = \infty$$

h 5. (2pts) Give numerical evidence that demonstrates the functions end behavior.

$\frac{101}{9894}$

Right

x	f(x)
100	.00981
10,000	1×10^{-3}
100,000	1×10^{-4}
1,000,000	1×10^{-5}

∞ 0

Left

x	f(x)
-100	$-\frac{101}{9894}$
-10,000	-1×10^{-4}
-100,000	-1×10^{-5}
-1,000,000	-1×10^{-6}

$-\infty$ 0

i. 7. Use limit notation to write end behavior limits, and then express each analytical limit as a sentence.

Right

$$\lim_{x\to\infty} f(x) = 0$$

left

$$\lim_{x\to-\infty} f(x) = 0$$

Fabian is cool!

8. Graph the function. Include all features and label all parts.