

f) **Vertical Asymptotes:** Give numerical evidence to demonstrate the functions behavior at all vertical asymptotes.

$$Y_1 = \frac{(2(x-3)(x+3))}{((x+1)(x+5)(x-3))}$$

Vertical Asymptote:  $x = -1$

x  
-1.5  
-1.25  
-1.1  
-1.01

Left Side	
x	f(x)
-1.5	
-1.25	
-1.1	
-1.01	

↓  
-∞

x  
-0.5  
-0.75  
-0.9  
-0.9999

Right Side	
x	f(x)
-0.5	
-0.75	
-0.9	
-0.9999	

↓  
∞

Vertical Asymptote:  $x = -5$

x  
-5.4  
-5.2  
-5.01  
-5.003

Left Side	
x	f(x)
-5.4	-2.727
-5.2	-5.23
-5.01	100.2
-5.003	-333.6

↓  
-∞

x  
-4.9  
-4.99  
-4.999  
-4.9999

Right Side	
x	f(x)
-4.9	9.7436
-4.99	99.749
-4.999	999.75
-4.9999	9999.7

-5.4  
-5  
∞

← -5 →

g) **Vertical Asymptotes:** Use limit notation to write left and right hand limits that describe the functions behavior near each vertical asymptote.

Vertical Asymptote:  $x = -1$

$$\lim_{x \rightarrow -1^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -1^+} f(x) = \infty$$

Vertical Asymptote:  $x = -5$

$$\lim_{x \rightarrow -5^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -5^+} f(x) = \infty$$

i) **End Behavior Asymptotes:** Use limit notation to write left and right end

Left

behavior limits.

Right

~~$$\lim_{x \rightarrow 0^-} f(x) = -\infty$$~~

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

$$\lim_{x \rightarrow +\infty} f(x) = 0$$

-0.0001

h) End Behavior Asymptotes: Give numerical evidence that demonstrates the functions right and left end behavior.

End Behavior Asymptote = Horizontal or Slant Asymptote:  $y=0$

Left End	
x	f(x)
-15,000	$-1 \times 10^{-4}$
-25,000	$-8 \times 10^{-5}$
-730,000	$-3 \times 10^{-6}$
-1,000,000	$-2 \times 10^{-6}$

Left end behavior: As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow 0$ .

Right End	
x	f(x)
15,000	$1.3 \times 10^{-4}$
25,000	$8 \times 10^{-5}$
730,000	$2.7 \times 10^{-5}$
1,000,000	$2 \times 10^{-6}$

Right end behavior: As  $x \rightarrow \infty$ ,  $f(x) \rightarrow 0$ .

Zeros	N, D, B	factor	Mult.	x-int V.A. R.D.	point or eq.	• ○ ↑
3	B	$(x-3)$	1	R.D.	$(3, 3/8)$	○
-3	N	$(x+3)$	1	x-int	$(-3, 0)$	•
-1	D	$(x+1)$	1	V.A.	$x = -1$	↑
-5	D	$(x+5)$	1	V.A.	$x = -5$	↑

j) Graph the function. Include all features and label all parts.

