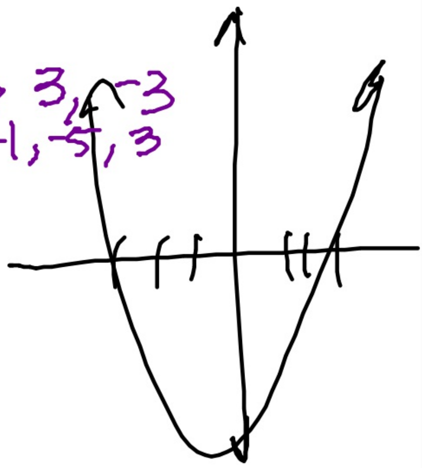
Vertical asymptotes, removable discontinuities and x-intercepts:

Factor the denominator and numerator into irreducible prime factors and find the zeros.

$$f(x) = \frac{2(x^2-9)}{(x+1)(x+5)(x-3)} = \frac{2(x-3)(x+3)}{(x+1)(x+5)(x-3)}.$$

zeros → 3, -3

zeros → -1, -5, 3



Handout: **Analyzing a Rational Function**

$$f(x) = \frac{2(x^2 - 9)}{(x+1)(x+5)(x-3)}$$

a) **End behavior asymptote:** Compare *n* and *m*.

LO: The degree of the numerator, *n*, is __2__ and the degree of the denominator, *m*, is __3__, because these are the __largest__ powers of *x* for the respective polynomial functions. Since __$n < m$__, we can conclude that the end behavior asymptote is the __horizontal asymptote__, __$y = 0$__, that means as x approaches __$\pm\infty$__ y approaches__0__.

Vertical asymptotes, removable discontinuities and x-intercepts:

b. Removable discontinuities or holes occur when the numerator and denominator share common factors that can be canceled entirely from the denominator. Find both the x-values and the y-values for every removable discontinuity = hole, and write as a point.

.375 → FRAC $\frac{3}{8}$

$$f(x)=\frac{2(x^2-9)}{(x+1)(x+5)(x-3)}=\boxed{\frac{2(x-3)(x+3)}{(x+1)(x+5)(x-3)}}.$$

zeros → 3, -3

zeros → -1, -5, 3

$x = 3$

$y = f(3) = \frac{3}{8}$

$y_1 =$

$y_2 = \frac{2(x+3)}{(x+1)(x+5)}$

$x=5$ $\quad \frac{2(5-3)(5+3)}{(5+1)(5+5)(5-3)} = \frac{2\cdot \cancel{2}\cdot 8}{6\cdot 10\cdot \cancel{2}}$

$x=3$ $\quad \frac{2\cancel{(3-3)}(3+3)}{(3+1)(3+5)\cancel{(3-3)}} = \frac{0}{0}$

removable discontinuity = hole = (3 , $\frac{3}{8}$)

$\frac{12}{32} = \frac{3}{8}$

Vertical asymptotes, removable discontinuities and x-intercepts:

c) The x-intercepts occur at the zeros of the numerator that are not also zeros of the denominator.

R.D.

$$f(x)=\frac{2(x^2-9)}{(x+1)(x+5)(x-3)}=\frac{2(x-3)(x+3)}{(x+1)(x+5)(x-3)}.$$

zeros $\rightarrow$ $\cancel{3}$, -3

zeros $\rightarrow$ -1, -5, $\cancel{3}$

$x-$ intercept $=($ -3 , 0 $)$

$$\frac{2(-3-3)(-3+3)}{\quad} = 0$$

Vertical asymptotes, removable discontinuities and x-intercepts:

d) The vertical asymptotes occur at the zeros of the denominator that can not be completely canceled from the denominator.

R.D.

$$f(x) = \frac{2(x^2-9)}{(x+1)(x+5)(x-3)} = \frac{2(x-3)(x+3)}{(x+1)(x+5)(x-3)} \begin{matrix} \rightarrow \cancel{3}, -3 \\ \rightarrow -1, -5, \cancel{3} \end{matrix}$$

A vertical asymptote of $f(x)$ is the vertical line $x =$ -1.

$x = -5.$

e) **y-intercept**: Find *f(0)*= 6/5.

The y-intercept of *f(x)* is the point (0, 6/5).

$$f(0) = \frac{2(0-3)(0+3)}{(0+1)(0+5)(0-3)} = \frac{2(\cancel{-3})(3)}{1 \cdot 5 \cdot (\cancel{-3})} = \frac{6}{5}$$

1.2

table

f) **Vertical Asymptotes:** Give numerical evidence to demonstrate the functions behavior at all vertical asymptotes.

$$\frac{2(x-3)(x+3)}{(x+1)(x+5)(x-3)}$$

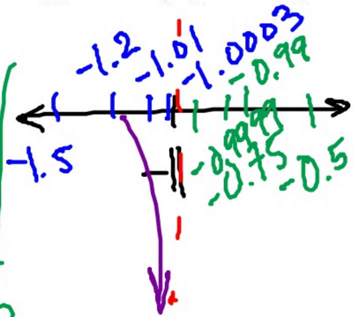
Vertical Asymptote: x = -1

x left → -1 | → $-\infty$

Left Side	
x	f(x)
-1.5	-1.74
-1.2	-4.737
-1.01	-99.75
-1.0003	-3333

x right → -1 | → ∞

Right Side	
x	f(x)
-0.5	2.2222
-0.75	4.2353
-0.99	100.25
-0.9999	10,000



Vertical Asymptote: x = -5

x left → -5 | → $-\infty$

Left Side	
x	f(x)
-5.5	−
	−
↓	−
-5.001	−

x right → -5 | → ∞

Right Side	
x	f(x)
-4.5	+
	+
	+
-4.9999	+

g) Vertical Asymptotes: Use limit notation to write left and right hand limits that describe the functions behavior near each vertical asymptote.

Vertical Asymptote: $x = -1$

$$\lim_{x \to -1^-} f(x) = -\infty \qquad \lim_{x \to -1^+} f(x) = \infty$$

Vertical Asymptote: $x = -5$

$$\lim_{x \to -5^-} f(x) = -\infty \qquad \lim_{x \to -5^+} f(x) = \infty$$