Graph a Rational Function (refer to handout)

The graph of $y = f(x)/g(x) = (a_n x^n + ...)/(b_m x^m + ...)$ has the following characteritics:

a. End behavior asymptote : Let n be the degree of the numerator polynomial and m the degree of the denominator polynomial.

If $n < m$, the end behavior asymptote is the horizontal asymptote $y = 0$.

If $n = m$, the end behavior asymptote is the horizontal asymptote $y = a_n / b_m$.

If $n > m$, the end behavior asymptote is the quotient polynomial function $y = q(x)$, where $f(x) = g(x)q(x) + r(x)$. There is no horizontal asymptote.

$\frac{a_n}{b_m}$

degree of numerator < degree of denominator

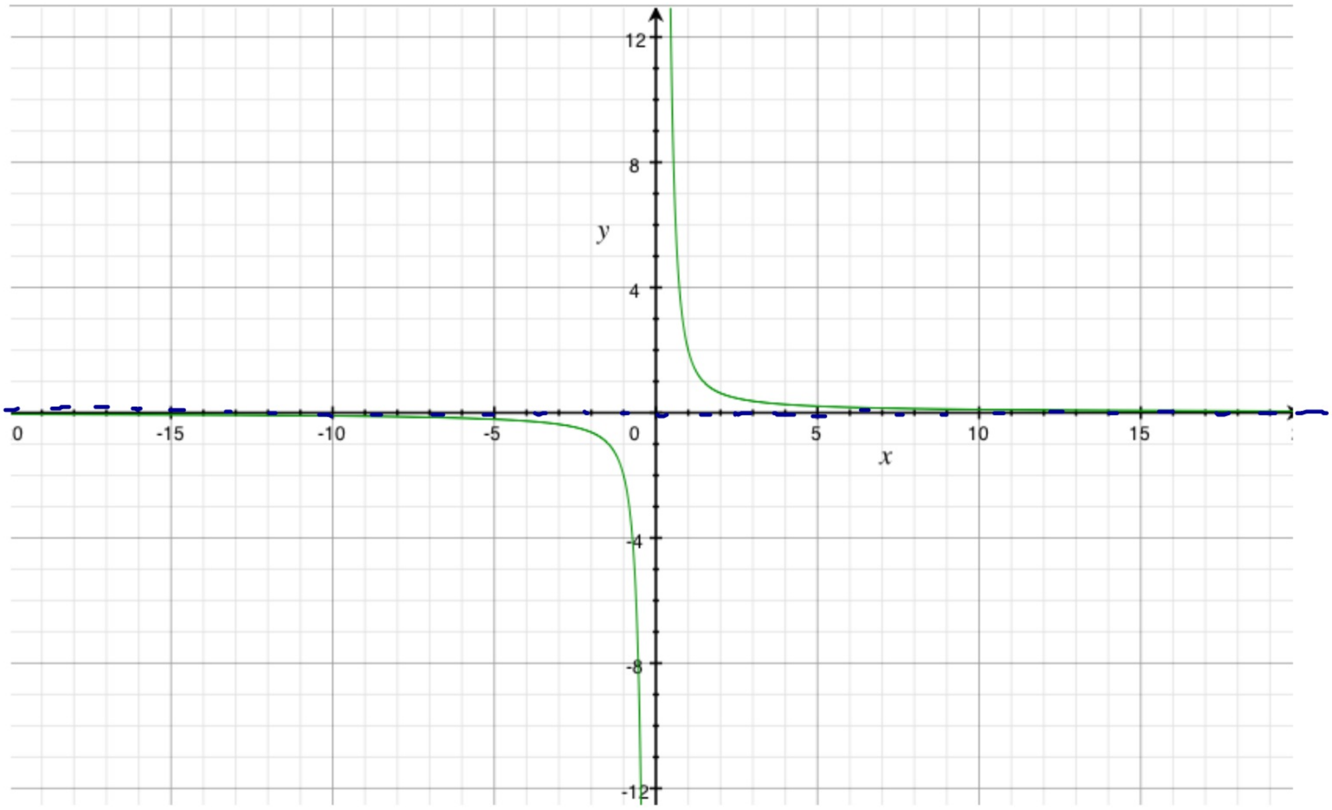
$$f(x) = \frac{x^2 + 1}{x^3}$$

$\frac{x^2}{x^3}$ $\frac{150^2}{150^3} = \frac{22,500}{3,375,000}$

$n = 2$

$m = 3$

LO: The degree of the numerator, *n,* is ___2___ and the degree of the denominator, *m,* is ___3___, because these are the ___largest___ powers of *x* for the respective polynomial functions. Since *n* < *m,* we can conclude that the end behavior asymptote is the horizontal line y=0, that means as x approaches ___$\pm\infty$___ y approaches___0___.



12
8
y
4
0
-15
-10
-5
0
5
10
15
x
-4
-8
-12

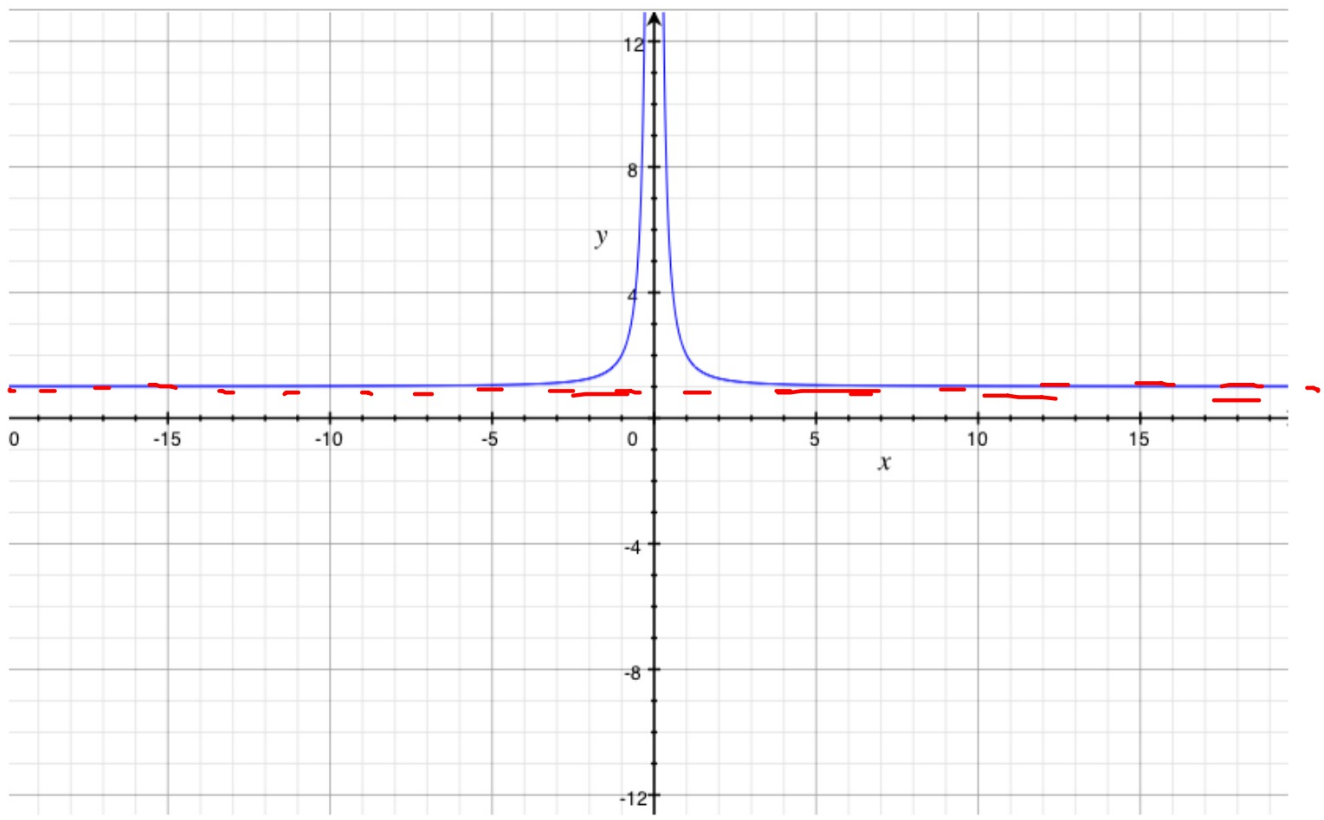
degree of numerator = degree of denominator

$$f(x) = \frac{(x^2 + 1)}{x^2} \Rightarrow \frac{1}{1} = 1 \Rightarrow \frac{301^2+1}{301^2} \quad \frac{90602}{90601}$$

$n = 2$

$m = 2$

LO: The degree of the numerator, *n,* is __2__ and the degree of the denominator, *m,* is __2__, because these are the __largest__ powers of *x* for the respective polynomial functions. Since *n* = *m,* we can conclude that the end behavior asymptote is the horizontal line y=~~a~~ $\frac{a_n}{b_m}$, that means as x approaches __$\pm\infty$__ y approaches__1__.



12
8
y
4
0
-15
-10
-5
0
5
x
10
15
-4
-8
-12

degree of numerator > degree of denominator

$$f(x) = \frac{x^3 + 1}{x^2}$$

Use long division to find the slant asymptote.

$n = 3$

$m = 2$

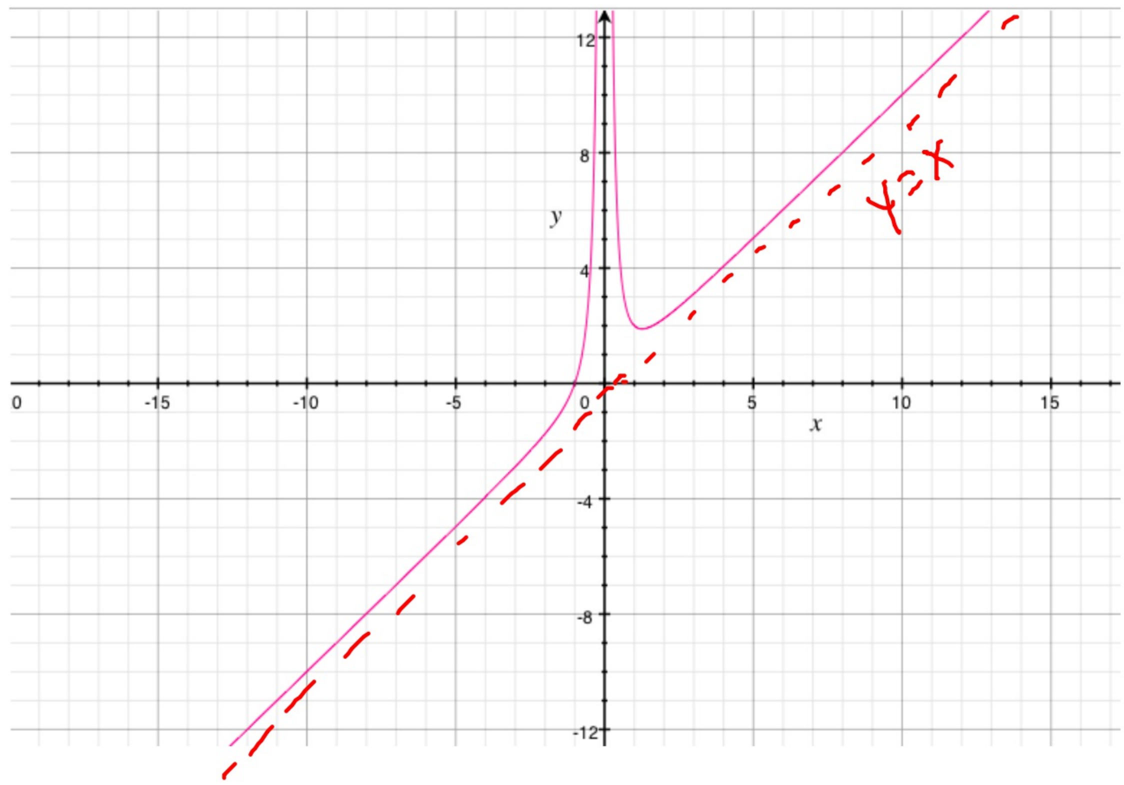
$$x^2 + 0x + 0 \;\overline{)\; x^3 + 0x^2 + 0x + 1}$$

$$-(x^3 + 0x^2 + 0x)$$

Quotient: x, remainder: 1

$$\frac{x^3}{x^2} = x$$

LO: The degree of the numerator, x^3+1, is 3 and the degree of the denominator, x^2, is 2, because these are the largest power of *x* for the respective polynomial functions. Since *n>m,* we can conclude that the end behavior asymptote is the quotient polynomial *q(x)* = x, that means as x approaches ∞ the *y* values of *f(x)* and *q(x)* approaches ∞ and as x approaches $-\infty$ the *y* values of *f(x)* and *q(x)* approaches $-\infty$; at the ends of the function *f(x)* looks and behaves like polynomial *q(x)*.


12
8
4
y
-4
-8
-12
0
-15
-10
-5
0
5
10
15
x
y=x

Graph a Rational Function (refer handout)

The graph of $y = \dfrac{f(x)}{g(x)}$ has the following characteritics:

b)Removable discontinuities : Zeros of the denominator provided that the zeros **are** also zeros of the numerator of equal or greater multiplicity. The corresponding factor can be cancelled entirely from the denominator and possibly the numerator.

c)x - intercepts : These occur at the zeros of the numerator, which are not also zeros of the denominator.

d) Vertical asymptotes : These occur at the zeros of the denominator, provided that the zeros **are not** also zeros of the numerator of equal or greater multiplicity. The corresponding factor can be cancelled entirely from the numerator.

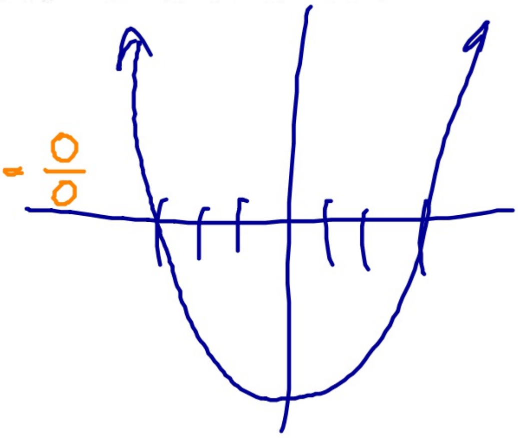
e) **y - intercepts :** This is the value of $f(0)$, if defined.

Vertical asymptotes, removable discontinuities and x-intercepts:

Factor the denominator and numerator into irreducible prime factors and find the zeros.

$$f(x) = \frac{2(x^2-9)}{(x+1)(x+5)(x-3)} = \frac{2(x+3)(x-3)}{(x+1)(x+5)(x-3)} = \frac{0}{0}$$

$x=-1 \quad x=-5 \quad x=3$



Handout: **Analyzing a Rational Function**

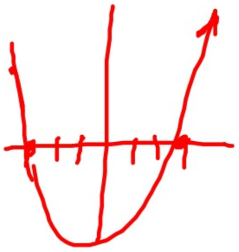
$$f(x) = \frac{2(x^2 - 9)}{(x+1)(x+5)(x-3)}$$ degree = 2, degree = 3

a) **End behavior asymptote:** Compare *n* and *m*.

LO: The degree of the numerator, *n,* is __2__ and the degree of the denominator, *m,* is __3__, because these are the __largest__ powers of *x* for the respective polynomial functions. Since __n < m__, we can conclude that the end behavior asymptote is the __horizontal asymptote__, __y = 0__, that means as x approaches __±∞__ y approaches __0__.

Vertical asymptotes, removable discontinuities and x-intercepts:

b. Removable discontinuities or holes occur when the numerator and denominator share common factors that can be canceled entirely from the function. Find both the *x*-values and the *y*-values for every removable discontinuity = hole, and write as a point.



$$f(x)=\frac{2(x^2-9)}{(x+1)(x+5)(x-3)}=\frac{2(x-3)(x+3)}{(x+1)(x+5)(x-3)}=\frac{2\cdot 2\cdot 8}{6\cdot 10\cdot 2}$$

$$\frac{2(3-3)}{0}$$

$x=$ 3

$y=f($3$)=$ 3/8

$$\frac{2(x+3)}{(x+1)(x+5)}$$

$$\frac{2(3+3)}{(3+1)(3+5)}$$

$x=3$ → 3/8

$\frac{12}{32}\rightarrow\frac{3}{8}$

removable discontinuity = hole = (3, 3/8)