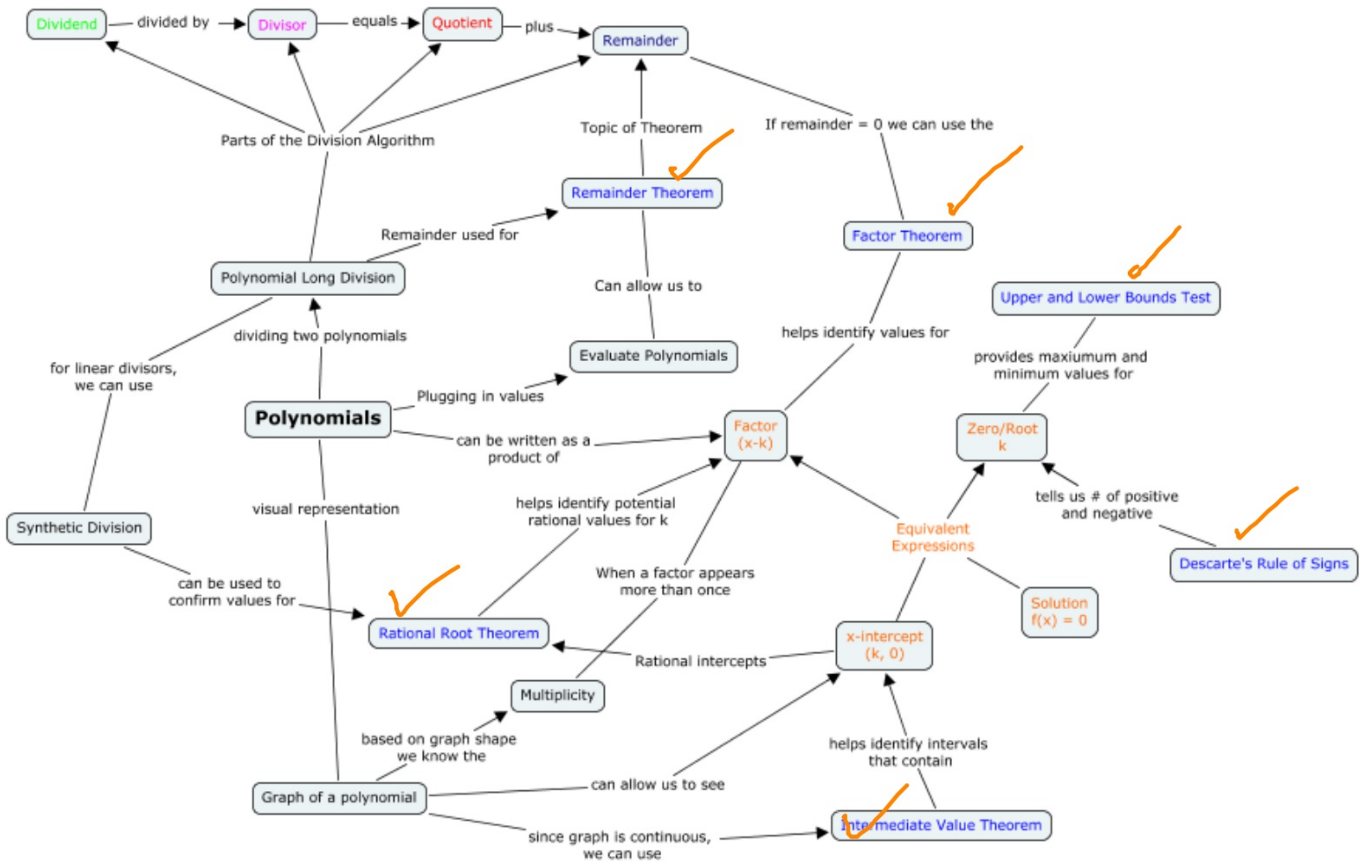

Dividend
divided by
Divisor
equals
Quotient
plus
Remainder
Parts of the Division Algorithm
Topic of Theorem
If remainder = 0 we can use the
Remainder Theorem
Factor Theorem
Remainder used for
Polynomial Long Division
Can allow us to
Upper and Lower Bounds Test
helps identify values for
dividing two polynomials
Evaluate Polynomials
provides maxiumum and minimum values for
for linear divisors, we can use
Polynomials
Plugging in values
Factor (x-k)
Zero/Root k
can be written as a product of
tells us # of positive and negative
visual representation
helps identify potential rational values for k
Synthetic Division
Equivalent Expressions
Descarte's Rule of Signs
When a factor appears more than once
can be used to confirm values for
Solution f(x) = 0
Rational Root Theorem
x-intercept (k, 0)
Rational intercepts
Multiplicity
based on graph shape we know the
helps identify intervals that contain
can allow us to see
Graph of a polynomial
Intermediate Value Theorem
since graph is continuous, we can use

Agree or Disagree

Polynomials 2.4 Edition

Multiplicity of a Zero of a Polynomial Function

If f is a polynomial function and $\left(x-c\right)^{m}$ is a factor of f but $\left(x-c\right)^{m+1}$ is not, then c is a zero of **multiplicity** $\boldsymbol{m}$ of f.

Remainder Theorem

If polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Factor Theorem

A polynomial function $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Find the remainder when $f(x) = 3x^2 + 7x - 20$ is divided by $x + 4$.

$f(-4) =$

If we can show that *f(-4)=0* is true , then we can claim that *(x-(-4))* = *(x+4)* is a factor of *f* by citing the factor theorem.

Fundamental Connections for Polynomial Functions

For a polynomial function f and a real number k, the following statements are equivalent:

1. $x = k$ is a solution of the equation $f(x) = 0$.

2. k is a zero (or root) of the function f.

3. $(k, 0)$ is an x-intercept of the graph of $y = f(x)$.

4. $x - k$ is a factor of $f(x)$

Rational Zeros Theorem

Suppose f is a polynomial function of degree $n \geq 1$ of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0$, with every coefficient an integer and $a_0 \neq 0$.

If $x = p / q$ is a rational zero of f, where p and q have no common integer factors other than 1, then

- p is an integer factor of the constant coefficient a_0, and

- q is an integer factor of the leading coefficient a_n.

Factors of a Polynomial with Real Coefficients

- **Theorem:** Every polynomial function (with real coefficients) can be uniquely factored into a product of linear factors and/or irreducible quadratic factors.

- **Corollary:** A polynomial function of odd degree has at least one real zero.

Upper and Lower Bound Tests for Real Zeros

Let f be a polynomial function of degree $n \geq 1$ with a positive leading coefficient. Suppose $f(x)$ is divided by $x - k$ using synthetic division.

• If $k \geq 0$ and every number in the last line is nonnegative (positive or zero), then k is an *upper bound* for the real zeros of f.

• If $k \leq 0$ and the numbers in the last line are alternately nonnegative and nonpositive, then k is a *lower bound* for the real zeros of f.

Intermediate Value Theorem (IVT)

- If a and b are real numbers with $a < b$ and if f is continuous on the interval $[a,b]$, then f takes on every value between $f(a)$ and $f(b)$.

- In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some number c in $[a,b]$.

Descartes' Rule of Signs

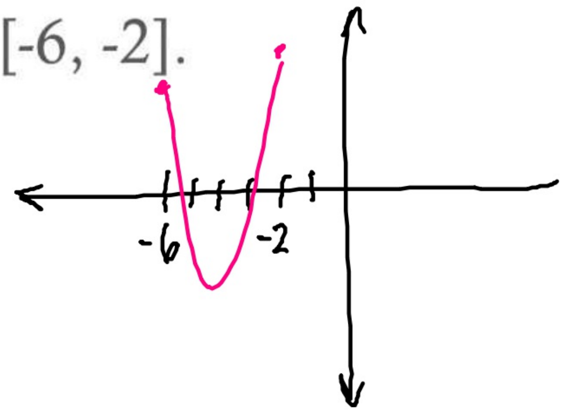
If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$
is a polynomial of degree n, then

1) The number of positive real zeros of f is equal to the number of variations in sign of $f(x)$, or that number less some even number.

2)The number of negative real zeros of f is equal to the number of variations in sign of $f(-x)$, or that number less some even number.

Agree or Disagree? Why?

- Given that f(-6) = 134 and f(-2) = 176, we can conclude there are no zeros on the interval [-6, -2].



Agree or Disagree?
Why?

- Given the polynomial $f(x)$ and $f(4) = 0$, we can conclude that $(x - 4)$ is a factor of the polynomial f.

Factor Thm.

Agree or Disagree? Why?

- The work below proves that 7 is a lower bound of the polynomial

$k \leq 0$

7 |	2	-18	41	-110	115
+		14	-28	91	-113
	2	-4	13	-19	2
	+	−	+	−	+

Agree or Disagree? Why?

- Given $f(x) = (x + 3)^4 (x - 1) (x+2)^{\cancel{3}}$, we know the following things are true:
  - $x = 1$ is a zero of $f(x)$
  - There is an x-intercept at the point (-3, 0), but the graph does not cross -even multiplicity
  - There is a solution of $x = -2$ to the equation $f(x) = 0$
  - The multiplicity of the factor $(x+2)$ is 3.

Agree or Disagree? Why?

- To prove that (x+2) is a factor of f(x), you could evaluate f(~~2~~).

$f(-2)=0.$

Agree or Disagree? Why?

- The fastest way to find a remainder to the division problem $\dfrac{x^{32}-27x^{15}+56x^{7}+13x^{4}-4x+8}{x-1}$ $= f(x)$ is to use synthetic division

Remainder Theorem $(x-k) \Rightarrow f(k) = r$

$k=1$

$f(1) = \text{remainder}$

Agree or Disagree? Why?

- Given the points on f(x): (-2, 0), $\boxed{(1, 72)}$ (3, 0), and (5, 0), the function f(x), can have the equation f(x) = (x + 2) (x – 3)(x - 5).

$f(1) = (1+2)(1-3)(1-5)\cdot 3 = 24\cdot 3$

$f(x) = 3(x+2)(x-3)(x-5)$

$\boxed{\text{Quiz 3}}$

a) $p(-1) = -2$ → $p(x)$ is continuous

$p(1) = 8$ → endpoints w/ opposite signs

→ where? $[-1, 1]$

b)

-6	1	2	5	3	-3
+	↓	-6	24	-174	1026
	1	-4	29	-171	1023
	+	−	+	−	+

→ alternating non-negative and nonpositive coefficients from synthetic division.

→ $k \leq 0$

c) $p(-x) = (-x)^4 + 2(-x)^3 + 5(-x)^2 + 3(-x) - 3$

$p(-x) = x^4 - 2x^3 + 5x^2 - 3x - 3$

$+ \quad - \quad + \quad - \quad -$

→ $p(-x) = ?$

→ # of sign changes (3)

→ how many negative real zeros 3 <u>or</u> 1