Descartes' Rule of Signs

If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$
is a polynomial of degree n, then

1) The number of positive real zeros of f is equal to the number of variations in sign of $f(x)$, or that number less some even number.

2)The number of negative real zeros of f is equal to the number of variations in sign of $f(-x)$, or that number less some even number.

Sentence Frames for Descartes Rule of Signs

We compared the signs of the coefficients of *f(x)* to determine that *f(x)* has _____sign changes in its coefficients, therefore ***f(x)*** has ____________________ positive real roots.

We found *f(-x)*=_________________ and compared the signs of the coefficients of *f(-x)* to determine that *f(-x)* has ______ sign changes in it's coefficients, therefore ***f(x)***_____________ negative real roots.

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Use Descartes' Rule of Signs to determine the possible number of positive and negative real zeros of the functions.

$f(x) = x^3 + x^2 - x + 1$

Descartes' Rule of Signs

Positive

$f(x) = x^3 + x^2 - x + 1$

$+ \quad + \quad - \quad +$

Negative

$f(-x) = (-x)^3 + (-x)^2 - (-x) + 1$

$-x^3 + x^2 + x + 1$

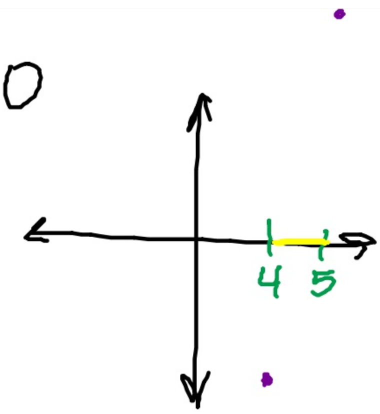
$- \quad + \quad + \quad +$

a) $p(x) = 3x^3 + x^2 - 62x + 20$

$[4, 5]$

$p(4) = -20$

$p(5) = 110$



Sentence Frames for IVT

Inconclusive

Since f(___) = _________ and f(___)= ________, the endpoints of the tested interval are both ________, that it is have the same sign and therefore the Intermediate Value Theorem **can not be used to make a conclusion about the zeros of f on** [_____,_____].

The function, *f*, is continuous on the interval, [4,5], and *f*(4) = -20 and *f*(5)= 110 since the *y*-values of the endpoints change from negative to positive on the tested interval, that is *f* has a sign change, we can conclude that there exists a zero, c such that f(c)=0 *. In other words *f* has a zero on the interval [4 , 5] by the Intermediate Value Theorem.

b) $p(x) = 3x^3 + x^2 - 62x + 20$

$x = 7 \qquad x = -6$

$$\begin{array}{r|rrrr} 7 & 3 & 1 & -62 & 20 \\ + & \downarrow & 21 & 154 & 644 \\ \hline & 3 & 22 & 92 & 664 \\ & + & + & + & + \end{array}$$

$$\begin{array}{r|rrrr} -6 & 3 & 1 & -62 & 20 \\ + & \downarrow & -18 & 102 & -240 \\ \hline & 3 & -17 & 40 & -220 \\ & + & - & + & - \end{array}$$

Sentence Frames for Upper and Lower Bounds Test

Lower bound: Since performing synthetic division on p(x) with k = -6 < 0 resulted in a quotient with alternating non-negative and non positive coefficients, k= -6 is a lower bound for the real zeros of p(x) * In other words, every real zero of p(x) must be greater then or equal to -6.

Upper bound: Since performing synthetic division on p(x) with k = 7 > 0 resulted in a quotient with all nonnegative coefficients k= 7 is an upper bound for the real zeros of p(x) * In other words, every real zero of p(x) must be less then or equal to 7.

Sentence Frames for Upper and Lower Bounds Test

Not a Lower bound: Since performing synthetic division on ______ with k = ____ < 0 **did not result** in a quotient with alternating __________________________ coefficients, k=____ is **not** a lower bound for the real zeros of ______.

Not an Upper bound: Since performing synthetic division on ______ with k = ____ > 0 **did not result** in a quotient with all positive coefficients k=____ is **not** an upper bound for the real zeros of ______.

c) $P(x) = 3x^3 + x^2 - 62x + 20$

$+ \quad + \quad - \quad +$

We compared the signs of the coefficients of *f(x)* to determine that *f(x)* has 2 sign changes in its coefficients, therefore ***f(x)*** has 2 or 0 positive real roots.

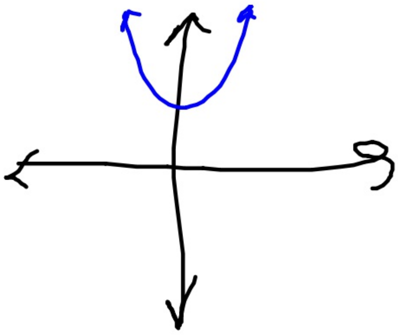
$P(-x) = 3(-x)^3 + (-x)^2 - 62(-x) + 20$

$-3x^3 + x^2 + 62x + 20$

$- \quad + \quad + \quad +$

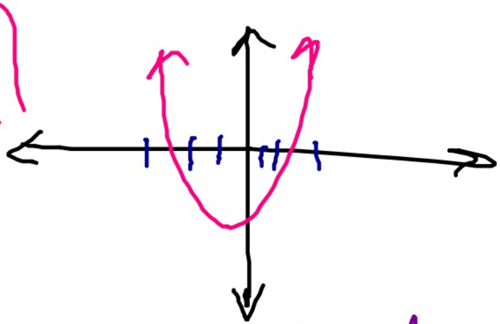
We found *f(-x)*= $-3x^3+x^2+62x+20$ and compared the signs of the coefficients of *f(-x)* to determine that *f(-x)* has 1 sign change~~s~~ in it's coefficients, therefore ***f(x)*** has 1 negative real root~~s~~.

$x^2+5=0$



* irreducible quadratic

$$\boxed{x^2-5=0}$$



$p=5$
$q=1$ $\pm 1, \pm 5$

$\Rightarrow$ irrational

$\sqrt{x^2}=\sqrt{5}$

$x=\pm\sqrt{5}$

quadratic formula

$(x+\sqrt{5})(x-\sqrt{5})$