Today's Objectives

Use the Intermediate Value Theorem and Descartes' Rule of Signs to determine the possible number of positive and negative real zeros and support your reasoning orally using key words, graphs and sentence frames.

Success Criteria

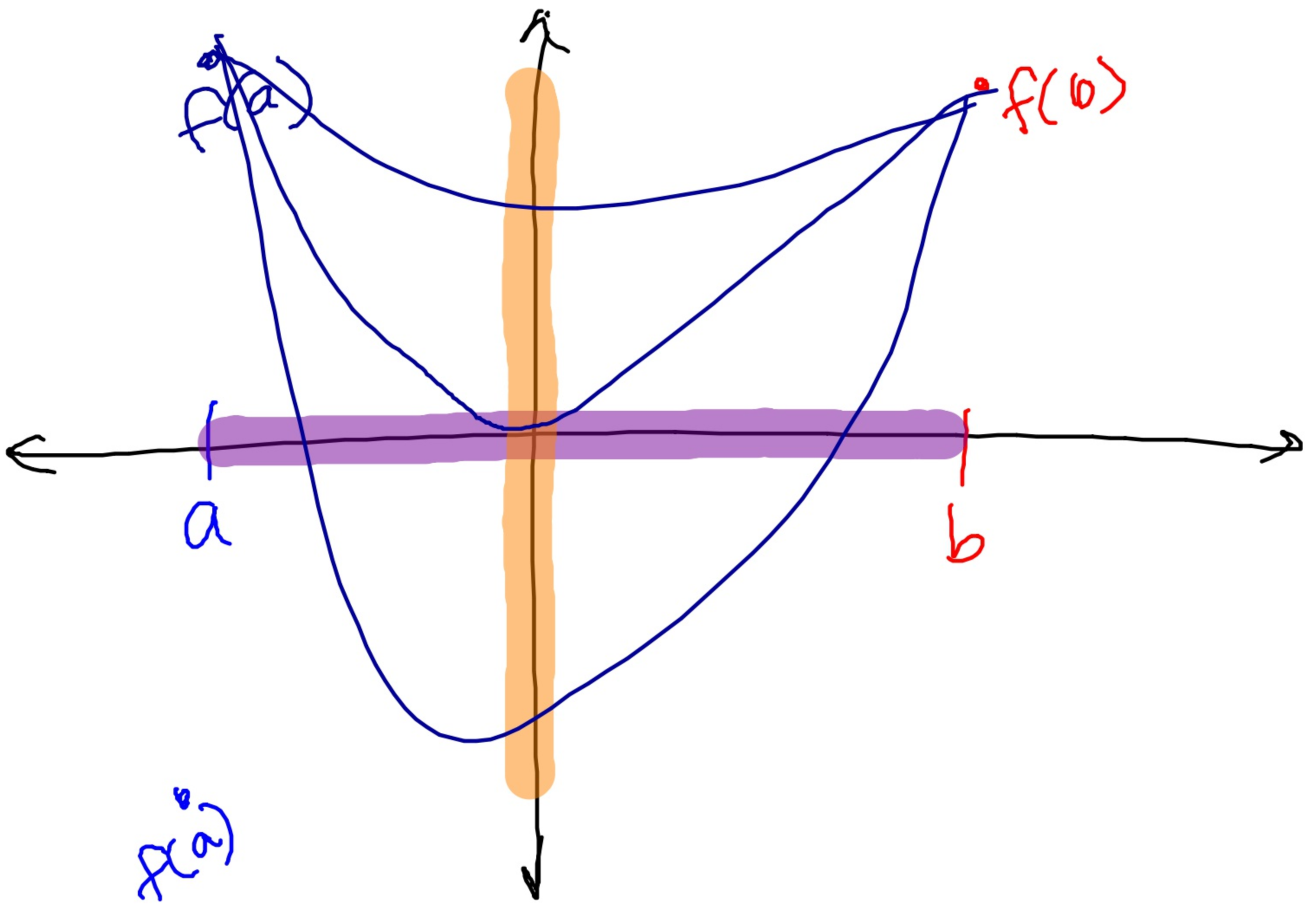
- Identify if the Intermediate Value Theorem applies to a problem situation.
- Define Descartes' Rule of Signs
- Identify sign changes for f(x)
- Evaluate function for f(-x)

Vocabulary: sign changes

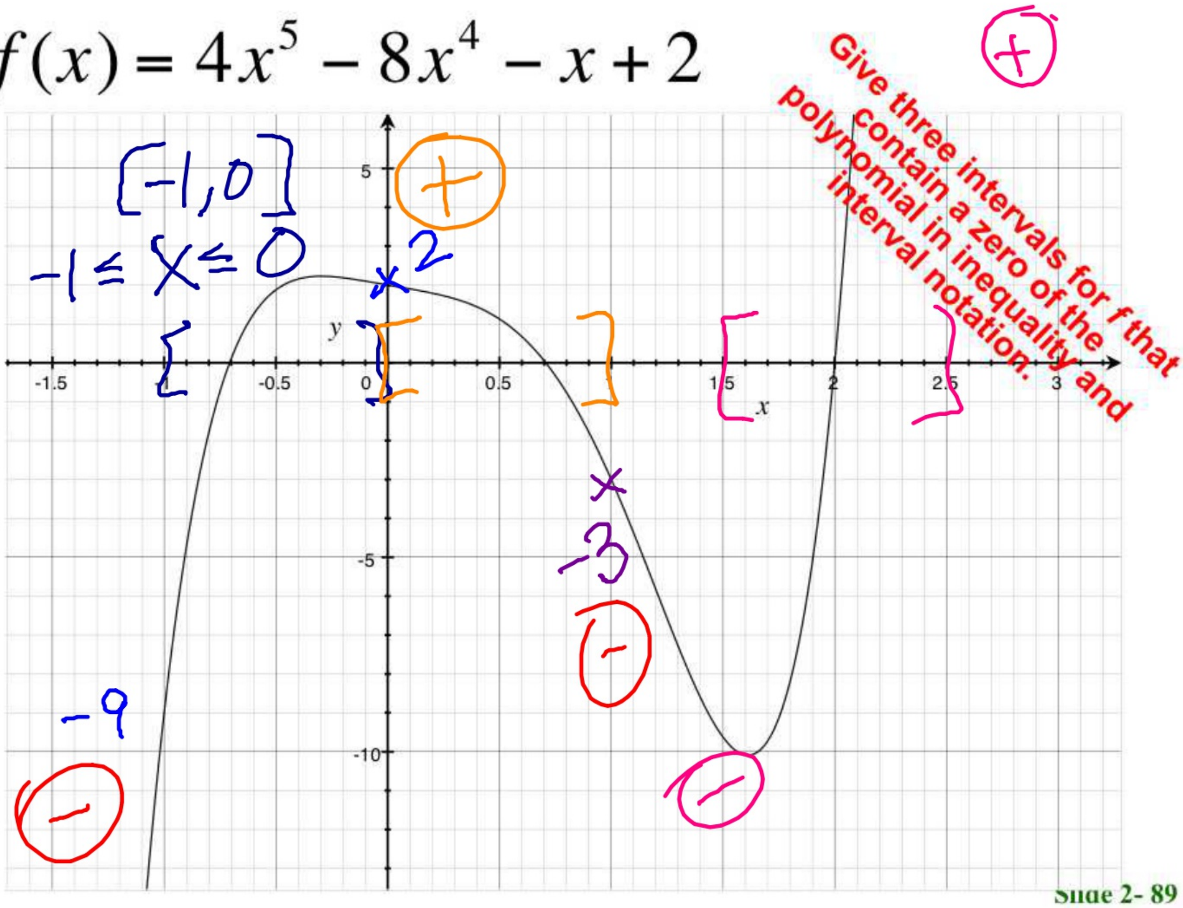
Intermediate Value Theorem (IVT)

- If a and b are real numbers with $a < b$ and if f is continuous on the interval $[a,b]$, then f takes on every value between $f(a)$ and $f(b)$.

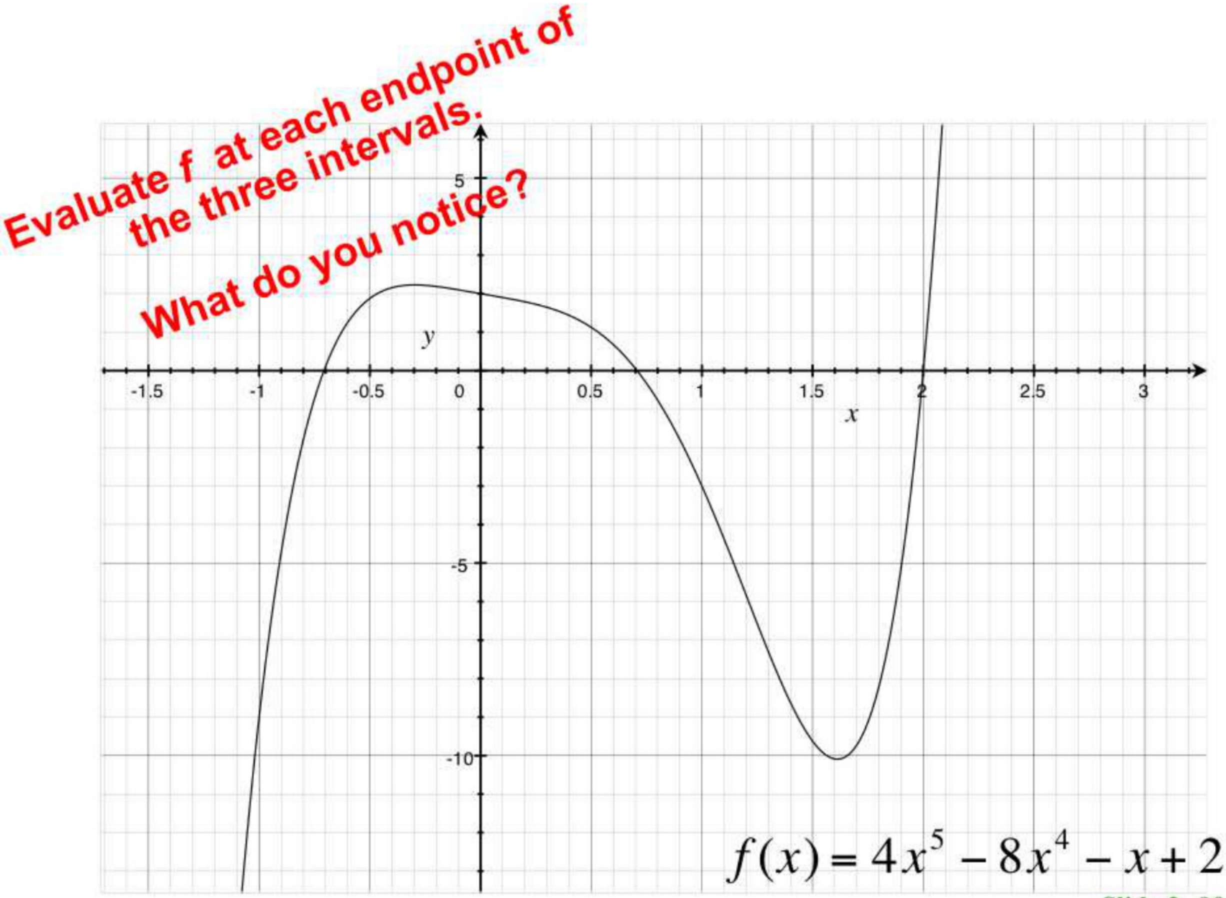
- In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0=f(c)$ for some number c in $[a,b]$.


f(a)
f(b)
a
b
f(a)

$$f(x) = 4x^5 - 8x^4 - x + 2$$



Give three intervals for f that contain a zero of the polynomial in inequality and interval notation.
[-1, 0]
-1 ≤ x ≤ 0
-9
2
-3
5
-5
-10
-1.5
-0.5
0
0.5
1.5
2
2.5
3
y
x


Evaluate f at each endpoint of
the three intervals.
What do you notice?
y
x
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
5
-5
-10

$$f(x) = 4x^5 - 8x^4 - x + 2$$

Sentence Frames for IVT

Since f(-8) = -2.1×10^5 and f(-7)= -1.1×10^5, the endpoints of the tested interval are both negative, that it is have the same sign and therefore the Intermediate Value Theorem **can not be used to make a conclusion about the zeros of f on** [-8, -7].

The function, *f*, is continuous on the interval, _______, and *f*(___) = _________ and *f*(___)= _________ since the *y*-values of the endpoints change from ___________ to ___________ on the tested interval, that is *f* has a sign change, we can conclude that there exists a ____________ such that ___________. In other words *f* has a zero on the interval [____, ____] by the Intermediate Value Theorem.

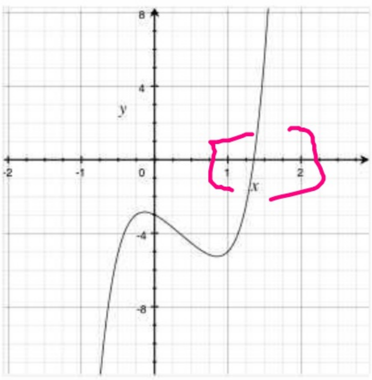
AM: Intermediate value Theorem

$f(x)=5x^5-9x^4+9x^3-5x^2-2x-3$

1. Use the intermediate value theorem to find the interval that contains an x-intercept of the function $f(x)=5x^5-9x^4+9x^3-5x^2-2x-3$.

[1, 2]

[A] $[-8,\ -7]$ [B] $[2,\ 3]$ [C] $[-3,\ -2]$ [D] none of these



A) $f(-8) = -2.1\times10^5$ (−)

$f(-7) = -1.1\times10^5$ (−)

B) $f(2) = 61$ (+)

$f(3) = 675$ (+)

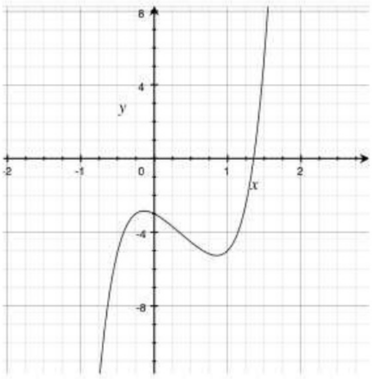
C) $f(-3) = -2229$

$f(-2) = -395$

12

1. Use the intermediate value theorem to find the interval that contains an x-intercept of the function $f(x) = 5x^5 - 9x^4 + 9x^3 - 5x^2 - 2x - 3$.

[A] $[-8, -7]$ [B] $[2, 3]$ [C] $[-3, -2]$ [D] none of these



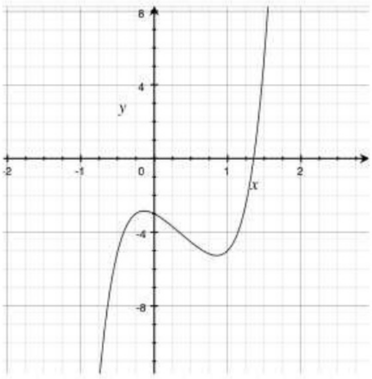
AM: Intermediate value Theorem

12

AM: Intermediate value Theorem

1. Use the intermediate value theorem to find the interval that contains an x-intercept of the function $f(x) = 5x^5 - 9x^4 + 9x^3 - 5x^2 - 2x - 3$.

[A] $[-8, -7]$ [B] $[2, 3]$ [C] $[-3, -2]$ [D] none of these



12

2. Use the Intermediate Value Theorem to show that the graph of the function has an x-intercept in the given interval. Approximate the x-intercept correct to two places.

$f(x) = x^3 + x^2 - 8x - 11$; $[-3, -2]$

AM: Intermediate value Theorem

12

AM: Intermediate value Theorem

$f(x) = x^2 - 6x - 12$
$-g(x) \quad -(-8x+36)$
$h(x) = x^2 + 2x - 48$

$h(-5) = -33 \quad h(4) = -24$

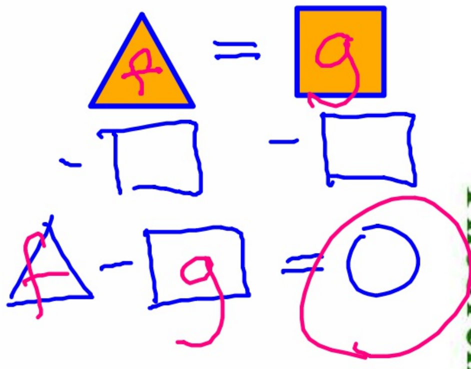
3. Use the intermediate value theorem to determine whether or not $f(x) = x^2 - 6x - 12$ and $g(x) = -8x + 36$ intersect on $[-5, 4]$. If applicable, find the point of intersection on the interval.

[A] $(f-g)(-5) = -33 < 0$ and $(f-g)(4) = -24 < 0$
There is a point of intersection at $(-4, 68)$.

[B] $(f+g)(-5) = 119 > 0$ and $(f+g)(4) = -16 < 0$
There is a point of intersection at $(6, -11)$.

[C] $(f-g)(-5) = -33 < 0$ and $(f-g)(4) = -24 < 0$
There is a point of intersection at $(3, 12)$.

[D] $(f-g)(-5) = -33 < 0$ and $(f-g)(4) = -24 < 0$
There is no point of intersection on $[-5, 4]$.



f – g is ____________ on the interval, *(f - g)(__)* = _________
and *(f - g)(__)*= ____

____

____

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