## Today's Objectives

- **Use synthetic division to find upper and lower bounds of polynomials** and write a justification using sentence frames and key words.
- Success Criteria
  - Define upper and lower bounds test for real zeros
  - Identify when upper and lower bounds test applies
- Vocabulary: upper bound, lower bound

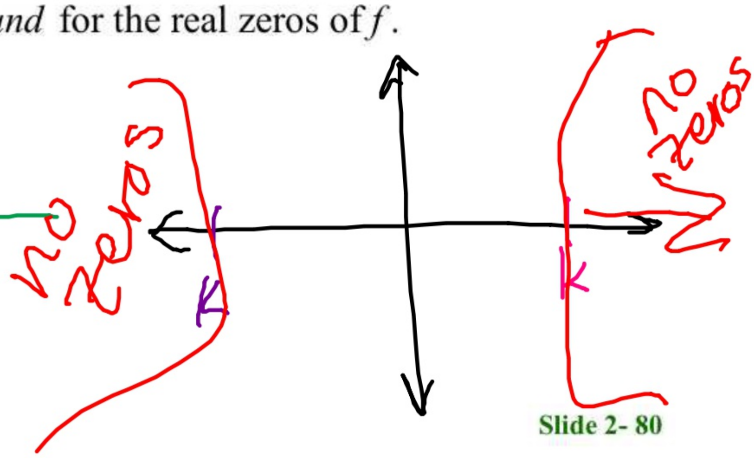
# Upper and Lower Bound Tests for Real Zeros

Let $f$ be a polynomial function of degree $n \geq 1$ with a positive leading coefficient. Suppose $f(x)$ is divided by $x - k$ using synthetic division.

+ or 0

- If $k \geq 0$ and every number in the last line is nonnegative (positive or zero), then $k$ is an *upper bound* for the real zeros of $f$.
- $\boxed{\text{If } k \leq 0}$ and the numbers in the last line are alternately nonnegative and nonpositive, then $k$ is a *lower bound* for the real zeros of $f$.

− or 0



# Sentence Frames for Upper and Lower Bounds Test

**Lower bound:** Since performing synthetic division on f(x) with k = -5 < 0 resulted in a quotient with alternating non-negative and non positive coefficients, k= -5 is a lower bound for the real zeros of f(x). In other words, every real zero of f(x) must be greater then or equal to -5.

**Upper bound:** Since performing synthetic division on f(x) with k = 8 > 0 resulted in a quotient with all nonnegative coefficients k= 8 is an upper bound for the real zeros of f(x). In other words, every real zero of f(x) must be less then or equal to 8.

# Sentence Frames for Upper and Lower Bounds Test

**Not a Lower bound:** Since performing synthetic division on f(x)__ with k = -3_ < 0 **did not result** in a quotient with alternating nonnegative and nonpositive coefficients, k= -3_ is **not** a lower bound for the real zeros of f____.

**Not an Upper bound:** Since performing synthetic division on _______ with k = ____ > 0 **did not result** in a quotient with all positive coefficients k=____ is **not** an upper bound for the real zeros of _______.

$f(x) = x^5 - 5x^4 - 20x^3 + 100x^2 + 64x - 322$

**1.** Use synthetic division to determine which pair of integers provide both a lower and an upper bound for the zeros of $f(x) = x^5 - 5x^4 - 20x^3 + 100x^2 + 64x - 322$.

[A] $-5, 8$ [B] $-3, 7$ [C] $-6, 7$ [D] $-4, 7$

| | | | | | | |
|---|---|---|---|---|---|---|
| $-5$ | 1 | $-5$ | $-20$ | 100 | 64 | $-322$ |
| + | | $-5$ | 50 | $-150$ | 250 | $-1570$ |
| | 1 | $-10$ | 30 | $-50$ | 314 | $-1892$ |
| | + | $-$ | + | $-$ | + | $-$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| 8 | 1 | $-5$ | $-20$ | 100 | 64 | $-322$ |
| + | | 8 | 24 | 32 | 1056 | 8960 |
| | 1 | 3 | 4 | 132 | 1120 | 8638 |

AM: Upper and Lower Bound Tests

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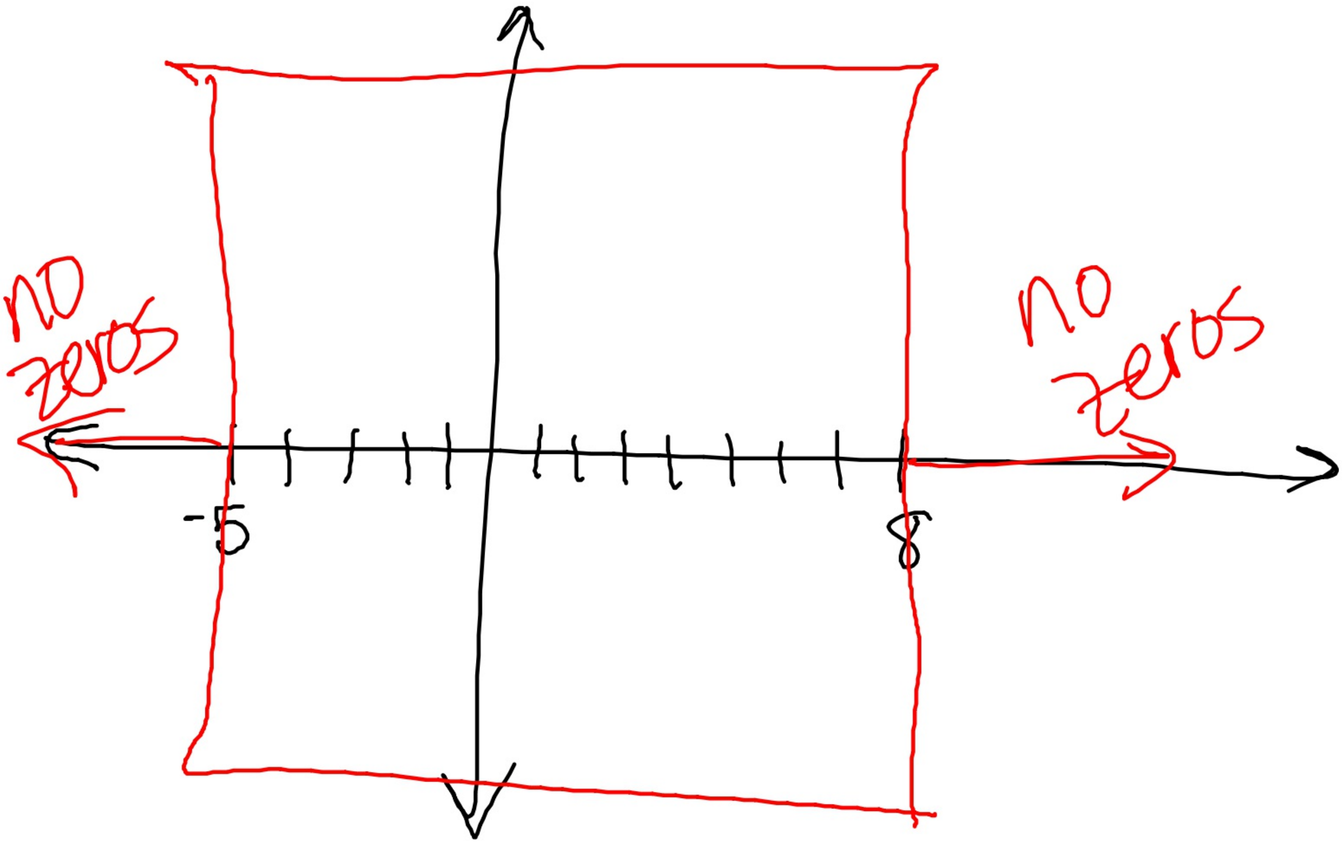
no zeros
-5
8
no zeros

$2x^4 - x^3 - 23x^2 + 46x - 24$

**2.** Use synthetic division to determine which pair of integers provide both a lower and an upper bound for the zeros of $f(x) = 2x^4 - x^3 - 23x^2 + 46x - 24$.

~~[A]~~ $-3, 5$ [B] $-4, 4$ ~~[C]~~ $-5, 3$ [D] none of these

$$\begin{array}{r|rrrrr} -4 & 2 & -1 & -23 & 46 & -24 \\ + & \downarrow & -8 & 36 & -52 & 24 \\ \hline & 2 & -9 & 13 & -6 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 4 & 2 & -1 & -23 & 46 & -24 \\ + & \downarrow & 8 & 28 & 20 & 264 \\ \hline & 2 & 7 & 5 & 66 & 240 \end{array}$$

AM: Upper and Lower Bound Tests

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