Today's Objectives

- **Use mathematical theorems** to produce the factored form of polynomials after listening to a step-by approach.
- Success Criteria
  - Use the factor theorem to find remainders of polynomial division
  - Use synthetic division to divide polynomials
  - Use the rational root theorem to identify possible zeros
  - Use equivalent expressions for roots, zeros, x-intercepts, and factors
- Vocabulary: root, zero, x-intercept, factor, solution

Factor Theorem

divides evenly $r(x)=0$

A polynomial function $f(x)$ has a factor $x-k$ if and only if $f(k)=0$.

Find the remainder when $f(x)=3x^2+7x-20$ is divided by $(x+4)$.

Factor

$f(-4) = 3(-4)^2+7(-4)-20=0$

$$\begin{array}{r|rrr} -4 & 3 & 7 & -20 \\ + & \downarrow & -12 & 20 \\ \hline & 3 & -5 & 0 \end{array}$$

If we can show that *f(-4)=0* is true , then we can claim that *(x-(-4))* = *(x+4)* is a factor of *f* by citing the factor theorem.

1. Use the factor theorem to determine which of the following is *not* a factor of $f(x) = 3x^4 + x^3 - 39x^2 + 23x + 12$.

[A] $3x - 2$ [B] $x - 3$ [C] $x + 4$ [D] $x - 1$

$2/3$ 3 -4 1

$$3x - 2 = 0$$
$$+2 \quad +2$$
$$\frac{3x}{3} = \frac{2}{3}$$
$$x = 2/3$$

Treat as 4 separate mini-problems, that ask you to check for a non-zero remainder when dividing *f(x),* by *3x-2, x-3, x+4,* and finally *x-1.* You **do not need to divide** to answer the question, use your calculator and the remainder theorem, with k = 2/3,3,-4,and finally 1. **How did I get these k values?**

AM: Factor Theorem

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AM: Factor Theorem

2. Use the factor theorem to determine how many of the following are factors of $f(x) = 4x^4 - 13x^3 - 80x^2 + 189x + 180$.

Possible factors: $4x+3$, $\cancel{x+4}$, $x-3$, $x+3$, $x-4$, $x+5$, $4x+9$

[A] 2 [B] 3 [C] 4 [D] 1

Treat as 7 separate mini-problems, that ask you to check for a zero remainder when dividing *f(x)*, by *4x+3, x+4, ….*

You **do not need to divide** to answer the question, use your calculator and the remainder theorem, with

k = $-3/4$, -4, 3, -3, 4, -5, $-9/4$.

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Rational Zeros Theorem

Suppose f is a polynomial function of degree $n \geq 1$ of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0$, with every coefficient an integer and $a_0 \neq 0$.

If $x = p / q$ is a rational zero of f, where p and q have no common integer factors other than 1, then

- p is an integer factor of the constant coefficient a_0, and
- q is an integer factor of the leading coefficient a_n.

Rational Zeros Theorem Problem #33, pg 224

$f(x) = 6x^4 - x^3 - 6x^2 - x - 12$

integer factors of the leading coefficient $\pm1, \pm2, \pm3, \pm6 \Rightarrow q$

integer factors of the constant term $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12 \Rightarrow p$

$\Rightarrow p$	$\Rightarrow q$	p/q			
±1	±1	±1	±4	±1	±4
±1	±2	±1/2	±4	±3	±4/3
±1	±3	±1/3	±6	±1	±6
±1	±6	±1/6	±12	±1	±12
±2	±1	±2			
±2	±3	±2/3			
±3	±1	±3			
±3	±2	±3/2			

How to Use a Graph to Approximate Zeros

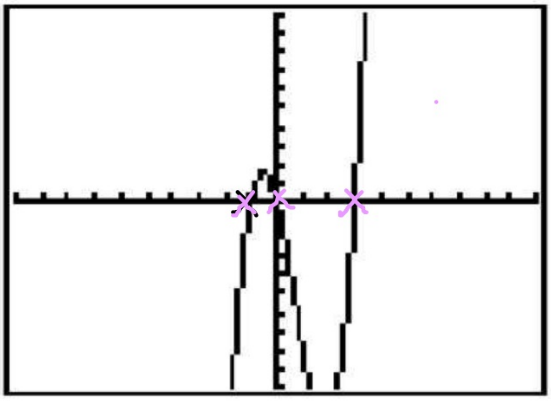
- 1. Graph the function in your calculator.
  - Be sure to set an appropriate view window
- 2. Find intersection points along the x-axis
- 3. Confirm values using an algebraic method or table values

Example **Finding the Zeros of a Polynomial Function**

Find the zeros of $f(x) = 2x^3 - 4x^2 - 6x$.

Using this graph, estimate the zeros of the function.

$$\begin{array}{r|rrrr} 3 & 2 & -4 & -6 & 0 \\ + & \downarrow & 6 & 6 & 0 \\ \hline & 2 & +2 & 0 & 0 \end{array}$$



List all of the potential rational zeros of the polynomial function. Do not attempt to find the zeros.

1. $f(x) = 3x^3 + 7x^2 + 4x - 15$

p: ±1, ±3, ±5, ±15
q: ±1, ±3

[A] $\pm 1,\ \pm 3,\ \pm 5,\ \pm 15,\ \pm\frac{1}{3},\ \pm\frac{5}{3}$

[B] $0,\ \pm 1,\ \pm 3,\ \pm 5,\ \pm\frac{1}{3},\ \pm\frac{5}{3}$

[C] $\pm 3,\ \pm 5,\ \pm 15,\ \pm 45,\ \pm\frac{1}{3},\ \pm\frac{5}{3}$

[D] $\pm 3,\ \pm 5,\ \pm 15,\ \pm\frac{1}{3},\ \pm\frac{5}{3},\ \pm\frac{7}{3}$

AM: Rational Zeros Theorem

LO: Possible rational zeros of the function are ___________.
From the possible rational zeros the actual zeros are _______________. I demonstrated this by using synthetic division and by applying the remainder theorem. This allowed me to show r = f(   ) = _____, r = f(   ) = _____, and r = f(   ) = _____.

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