Today's Objective

- Read a mathematical text using a reading guide in order to translate **between equivalent representations of zeros** and determine **the multiplicity of a zero** using key words.
- Success Criteria
  - Express zeros in terms of a solution to f(x)=0
  - Express zeros in terms of x-intercepts
  - Determine the factors of a polynomial
  - Identify the multiplicity of a zero
  - Determine whether an x-intercept crosses the x-axis
- Vocabulary: zero, factor, solution, x-intercept, multiplicity

Equivalent Representations of Zeros

Factor	x-intercept	Solution	Zero/Root
$(x+7)$	$(-7,0)$	$x=-7$	-7
$(x-4)$	$(4,0)$	$x=4$	4

Factor	x-intercept	Solution	Zero/Root
$(x-3/2)$	$(3/2,0)$	$x=3/2$	$3/2$
$(x+4/3)$	$(-4/3,0)$	$x=-4/3$	$-4/3$

$(x-3/2)$ $\quad 2x-3=0$

$\qquad\qquad\quad +3 \;\; +3$

$(2x-3)$ $\quad \frac{2x}{2}=\frac{3}{2}$

$(3x+4)$ $\quad x=3/2$

4. Use synthetic division to divide $x^4 - 9x^3 + 24x^2 - 20x$ by $x + 1$.

$$x^3 - 10x^2 + 34x - 54$$

$$x+1 \,\big)\, x^4 - 9x^3 + 24x^2 - 20x + 0$$

$$-(x^4 + x^3)$$

$$-10x^3 + 24x^2$$

$$-(-10x^3 - 10x^2)$$

$$34x^2 - 20x$$

$$-(34x^2 + 34x)$$

$$-54x + 0$$

$$-(-54x - 54)$$

$$54$$

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4. Use synthetic division to divide $x^4 - 9x^3 + 24x^2 - 20x$ by $x + 1$.

$x+1 \rightarrow -1$

-1	1	-9	24	-20	0
$+$	$\downarrow$	-1	10	-34	54
	1	-10	34	-54	54

$$x^3 - 10x^2 + 34x - 54 + \frac{54}{(x+1)}$$

$3x + 7$

$-7|3$

Remainder Theorem

$(x+1)$ $f(-1)$

If polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

What is this statement really telling you?

How can you use this information to verify your long division?

Example **Using the Remainder Theorem**

$2x^2 - x + 12$

Find the remainder when $f(x) = 2x^2 - x + 12$ is divided by $x + 3$. $R = 33$

$$
\begin{array}{r|rrr}
-3 & 2 & -1 & 12 \\
 & \downarrow & -6 & 21 \\
\hline
 & 2 & -7 & 33
\end{array}
$$

$= f(-3)$

$x^4+8x^3+6x^2-72x-135$

2. Use synthetic division to divide $x^4+8x^3+6x^2-72x-135$ by $x-4$.

$(x-4)$

[A] $x^3+12x^2+54x+144+\dfrac{441}{x-4}$ [B] $x^3+4x^2-10x-32$

[C] $x^3+12x^2+54x+144$ [D] $x^3+4x^2-10x-32-\dfrac{7}{x-4}$

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$$
\begin{array}{r|rrrrr}
4 & 1 & 8 & 6 & -72 & -135 \\
  & \downarrow & 4 & 48 & 216 & 576 \\
\hline
  & 1 & 12 & 56 & 144 & \boxed{441}
\end{array}
$$

$(x+5)$

1. Use synthetic division to find $f(-5)$ if $f(x) = 2x^6 - 49x^4 + 3x^3 - 14x^2 - 11x - 10$.

[A] −55 [B] −35 [C] −37,555 [D] none of these

$2x^6 - 49x^4 + 3x^3 - 14x^2 - 11x - 10$

$$\begin{array}{r|rrrrrrr} -5 & 2 & 0 & -49 & 3 & -14 & -11 & -10 \\ + & \downarrow & -10 & 50 & -5 & 10 & 20 & -45 \\ \hline & 2 & -10 & 1 & -2 & -4 & 9 & -55 \end{array}$$

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$(x+4)$

$\parallel$

$x^6+2x^5+3x^3-2x^2-33$

1. Use the remainder theorem to find $P(-4)$ if $P(x) = x^6 + 2x^5 + 3x^3 - 2x^2 - 33$. Find the quotient polynomial that leads to the remainder.

[A] 1791; $x^5 - 2x^4 + 8x^3 - 29x^2 + 114x - 456$

[B] 1864; $x^5 + 6x^4 + 8x^3 + 35x^2 - 118x - 456$

[C] 1864; $x^5 - 2x^4 + 8x^3 - 29x^2 + 114x - 456$

[D] 1791; $x^5 + 6x^4 + 8x^3 + 35x^2 - 118x - 456$

-4	1	2	0	3	-2	0	-33
	+						

LO: The value of ______ is equal to the remainder of the quotient P(x) divided by ________. By dividing P(x) by ________, I get the quotient ____________.

AM: Remainder Theorem

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