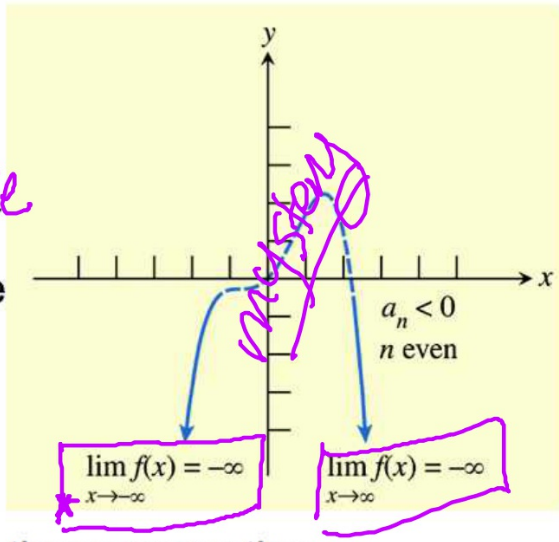
Leading Term Test for Polynomial End Behavior

For any polynomial function $f(x) = a_n x^n + \ldots + a_1 x + a_0$, the limits $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$ are determined by the degree n of the polynomial and its leading coefficient a_n:



The leading coefficient is negative because a_n $<$ 0.
This polynomial has even degree because n is even.
Examples of ______ numbers Include ______, ______, and ______.

Find two 4-term polynomials with these properties.

End Behavior Song

- http://www.youtube.com/watch?v=ohni1PVrlek&feature=share&list=FLbzBuPFkjwlDIwDqRBsotsA

Example **Applying Polynomial Theory**

Describe right end behavior of $g(x) = 2x^4 - 3x^3 + x - 1$ using limits.

$$\lim_{\boxed{x \to +\infty}} g(x) = \infty$$

right

x	$g(x)$
100	1.97×10^8
200	3.18×10^9
300	1.6×10^{10}
400	5.1×10^{10}
500	1.2×10^{11}
600	2.6×10^{11}
700	4.8×10^{11}

Describe the right hand end behavior of $g(x) = 2x^4 - 3x^3 + x - 1$ using limits.

$$\lim_{x \to \infty} g(x) = \infty$$

LO: The limit of the polynomial function *g(x)* is ___∞___ as *x* approaches ___∞___. I know this because when I look at a table of values for the function that has large positive x values in increasing order, the y values in the table are ___large___ and ___increasing___. This numerical evidence means that as x approaches ___∞___ y approaches ___∞___.

Example **Applying Polynomial Theory**

Describe left end behavior of $g(x) = 2x^4 - 3x^3 + x - 1$ using limits.

$$\lim_{x \to -\infty} g(x) = \infty$$

x	$g(x)$
-1000	2×10^{12}
-900	1.3×10^{12}
-800	8.2×10^{11}
-700	4.8×10^{11}
-600	2.6×10^{11}
-500	1.3×10^{11}
-400	5.1×10^{10}

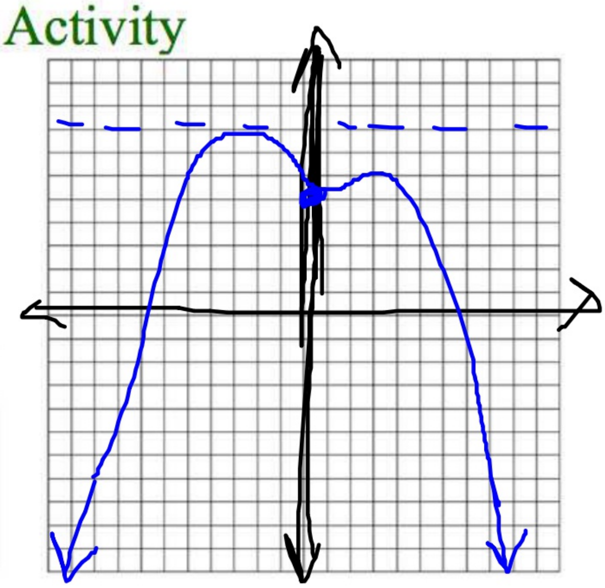
Describe the right hand end behavior of $g(x) = 2x^4 - 3x^3 + x - 1$ using limits.

$$\lim_{x \to -\infty} g(x) = \infty$$

LO: The limit of the polynomial function *g(x)* is __∞__ as *x* approaches __$-\infty$__. I know this because when I look at a table of values for the function that has large negative x values in decreasing order, the y values in the table are __large__ and __increasing__. This numerical evidence means that as x approaches __$-\infty$__ y approaches __∞__.

Graphing Activity

1. Let *p(x)* be a polynomial function with the following characteristics.
   - y-intercept of 5
   - End behavior limits are:
     $\lim_{x\to-\infty} p(x) = -\infty$ and $\lim_{x\to\infty} p(x) = -\infty$
   - upper bound of y = 8



a. Sketch a possible graph for *p(x)*.

b. Write a possible definition for *p(x)*. Note parts a. and b. do not need to be the same polynomials.

$$p(x) = -2x^4 - 3x + 5$$

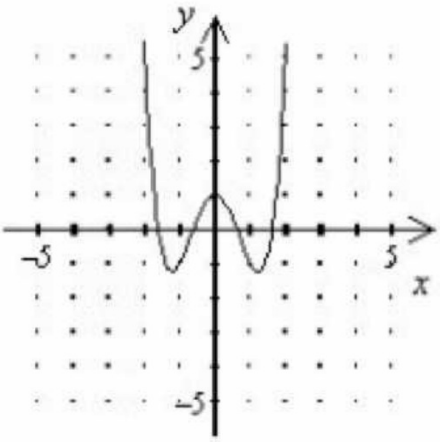
c. The right end behavior: As the input x approaches _____________, the output, y = f(x), approaches _______________.

d. The left end behavior: As the input, x, approaches _____________, the output, y = f(x), approaches _______________.

e. To show 5 is the y-intercept, we can evaluate $p(0) = -2(0)^4 - 3(0) + 5 = 5$ and we know that the graph must pass through the point (0,5) which is on the y axis or x = 0.

AM: Graph nth degree polynomials

1. Which function matches the graph?



LO: Since the graph has a W shape, it must be a polynomial of degree ________.

Since y goes to ___________ as *x* gets large and positive and since *y* goes to ____________ as *x* gets large and negative, the leading coefficient for *f(x)* must be _______ therefore *f(x)* =____________________.

[A] $f(x) = x^4 - 3x^2 + 1$

~~[B]~~ $f(x) = -x^5 - 3x^3 - 1$

[C] $f(x) = -x^4 + 3x^2 + x$

~~[D]~~ $f(x) = x^5 - 3x^3 + 1$

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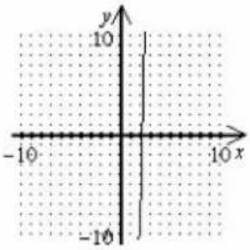
AM: Graph nth degree polynomials

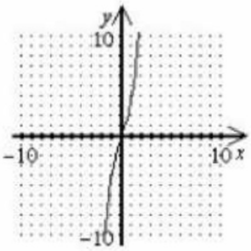
+ odd

2. Graph: $y = 3x^3 + 2x^2 - 9x - 18$

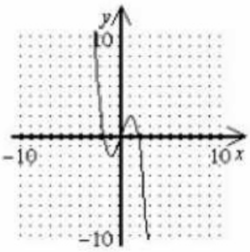
[A]



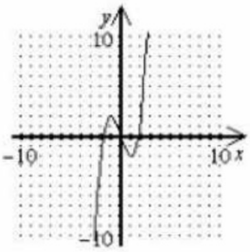
~~[B]~~



~~[C]~~



~~[D]~~



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